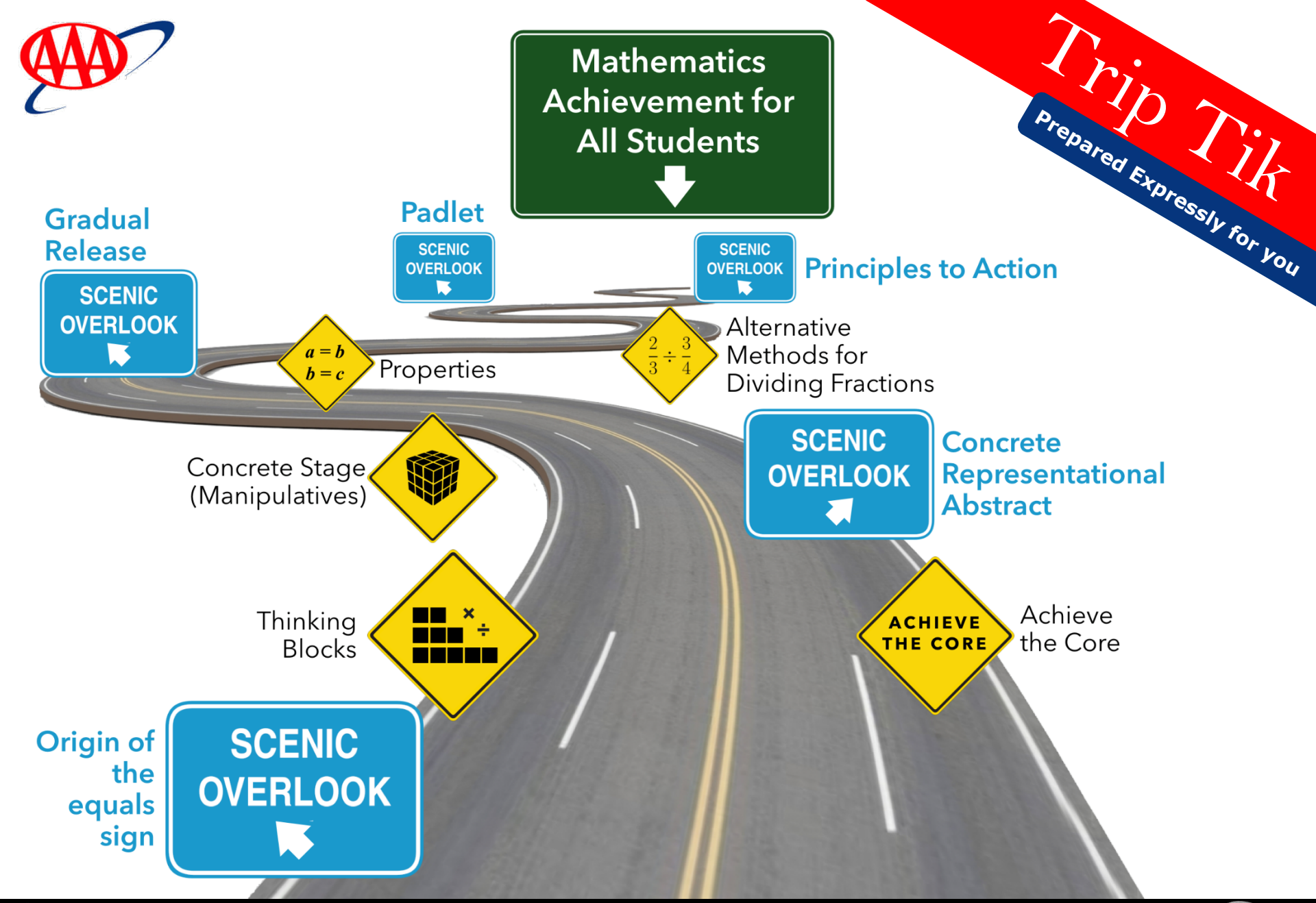
Solving Word Problems by Looking for Equality: Tape Diagrams Across Grade Levels

Mattoon's MEANING 4 MEMORY


Trip Tik
Prepared Expressly for you
Mathematics Achievement for All Students
Gradual Release
SCENIC OVERLOOK
Padlet
SCENIC OVERLOOK
SCENIC OVERLOOK
Principles to Action
a = b
b = c
Properties
2/3 ÷ 3/4
Alternative Methods for Dividing Fractions
Concrete Stage (Manipulatives)
SCENIC OVERLOOK
Concrete Representational Abstract
Thinking Blocks
ACHIEVE THE CORE
Achieve the Core
Origin of the equals sign
SCENIC OVERLOOK

Scenic Overlook: Origin of the Equals Sign

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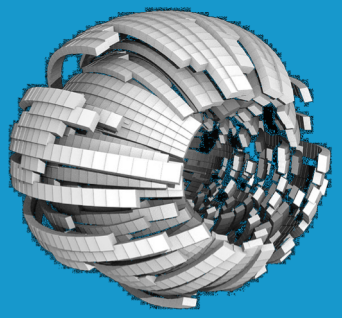


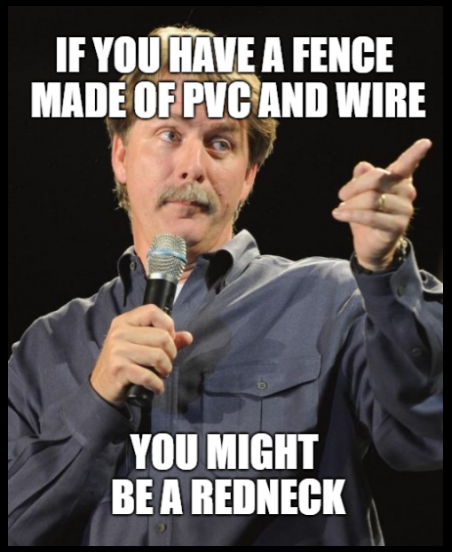
What about the equals sign?

Where did the equal sign come from?

What does it represent?

How does it demonstrate equivalence?

Real Life Word Problem: My Junky Fence



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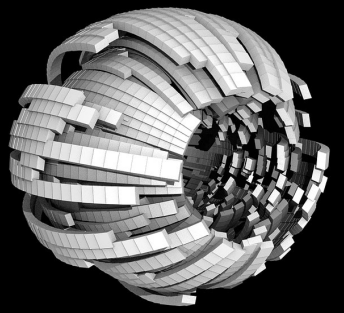
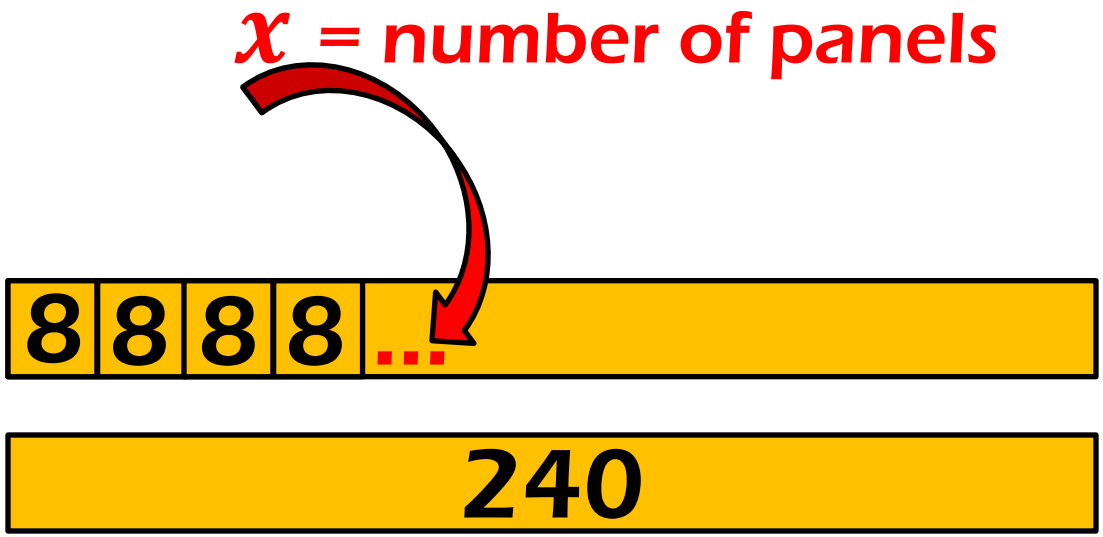
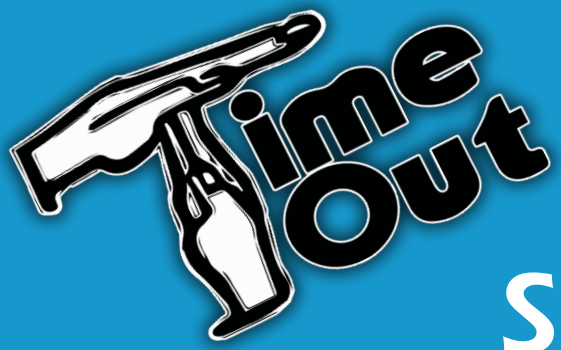


Real Life & Word Problems

Many real life problems and word problems, especially algebraic word problems, are grounded in equivalence.

How many eight foot long wood picket panels do I need to replace my junky fence that is 240 feet long?





Scenic Overlook: Concrete – Representational – Abstract (CRA)

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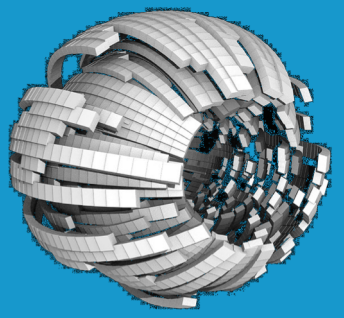


Levels of Abstraction = Concreteness Fading
Concrete, Representational, Abstract

During the first half of my teaching career, I would spend what seemed to be the first half of a math lesson teaching a new math concept by **sharing definitions, formulas, steps and procedures.**

To make things more challenging for my students, **I would simultaneously introduce the symbolic notation** used to represent those ideas. Then, I would spend the remainder of the lesson **attempting** to help my students make sense of these very **new** and often **abstract ideas**.

By the end of the lesson, I could help many students build an understanding, but **there was always a group I felt who I would leave behind.** Like many other teachers, I was just **teaching in a very similar way to that how I was taught**. **I knew no different.**

However, if we consider that **new learning requires the linking of new information with information they already know and understand**, we should be intentionally planning our lessons with this in mind. A great place to start new learning is through the use of a **meaningful context** and **utilizing concrete manipulatives** that students can touch and feel.

When we teach in this way, we **minimize the level of abstraction** so students can **focus their working memory on the new idea** being introduced in a meaningful way.

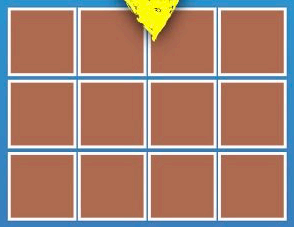
- Kyle Pierce, Tap into Teen Minds, https://tapintoteenminds.com/concreteness-fading/

Concreteness Fading

How many donuts are in 4 boxes of 12 donuts?

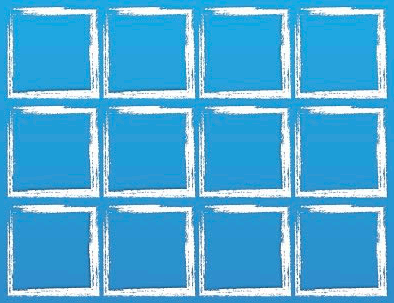
1 Concrete

Actual Doughnuts



Concrete Manipulatives

2 Visual



Drawings and Diagrams

3 Abstract

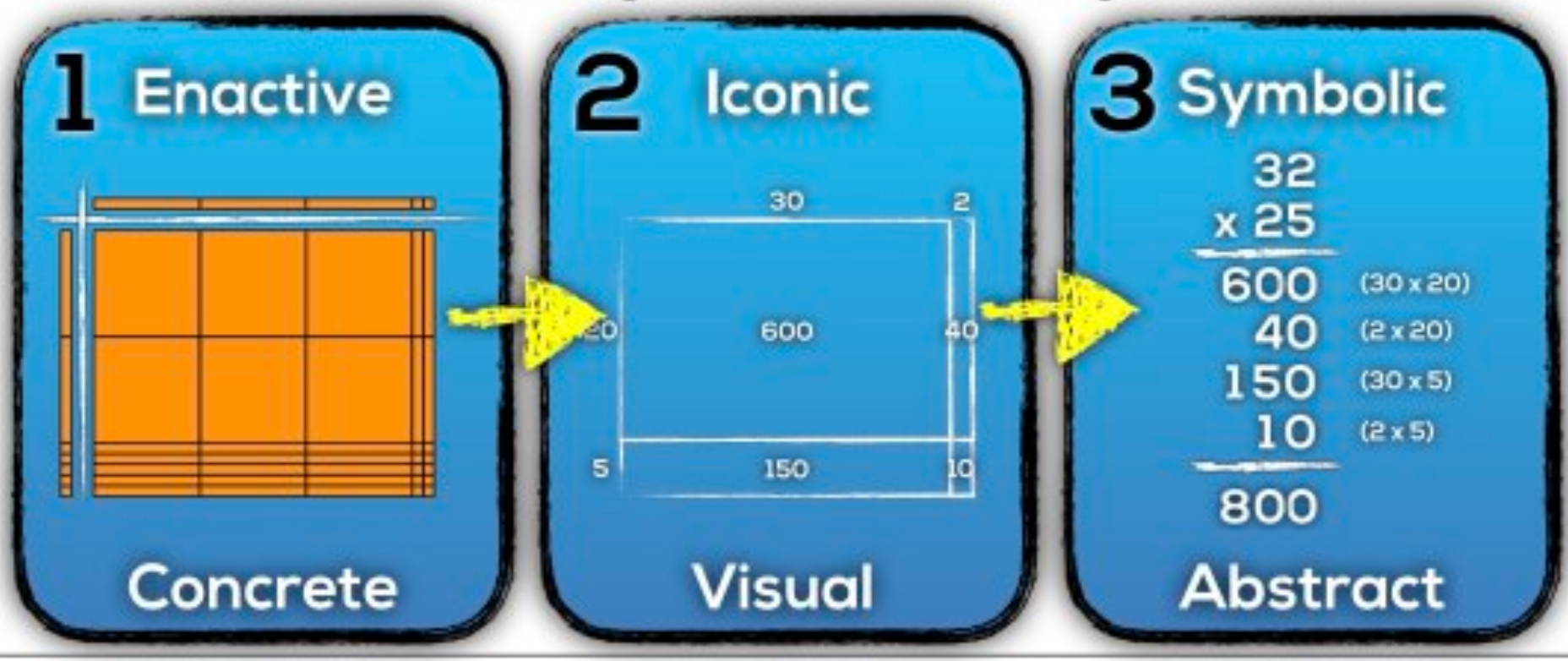
3 groups of
4 doughnuts
is equal to
12 doughnuts

Symbolic

3 x 4 = 12

Concreteness Fading

How many doughnuts are in the giant box?



TAPINTOTEENMINDS.COM @MATHLETEPEARCE

Jerome Bruner (1966) proposed three modes of representation:

Enactive representation (action-based)

Iconic representation (image-based)

Symbolic representation (language-based)

First Grade: Two Step Problems with CRA

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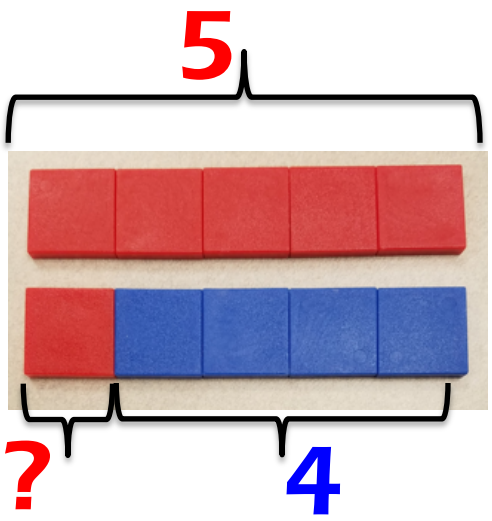
Concrete Stage (Manipulatives)

Mrs. Jordan's class borrowed 5 books from the library. They returned 4 books. Later, the class borrowed 2 new books. How many books did Mrs. Jordan's class have then?

Step 1



Step 2

The students' manipulation of the tiles acts as a formative assessment and is an avenue into their thinking allowing for true scaffolding to take place.

Representational Stage (to scale)

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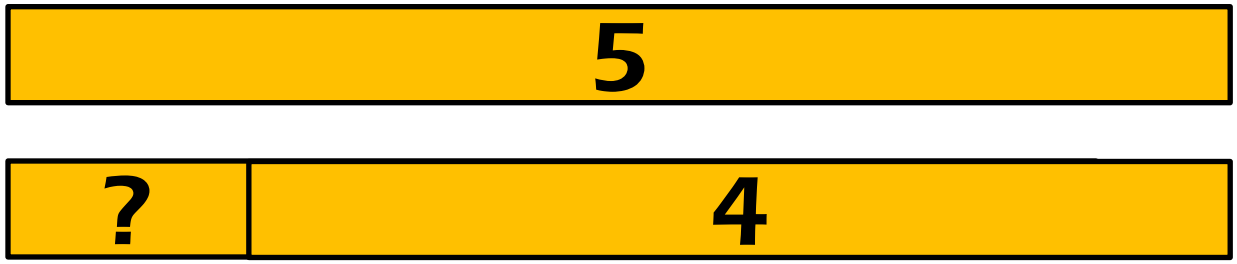
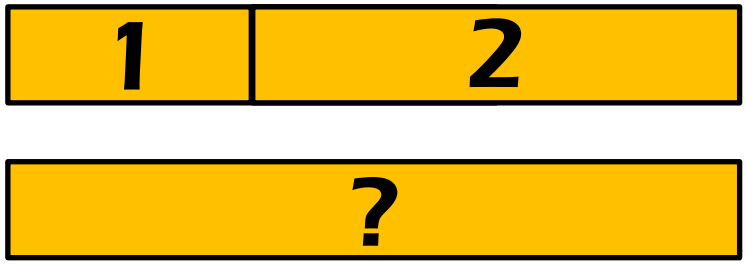


Step 2



These boxes, tape diagrams, are to scale.

Connecting to the Abstract Stage

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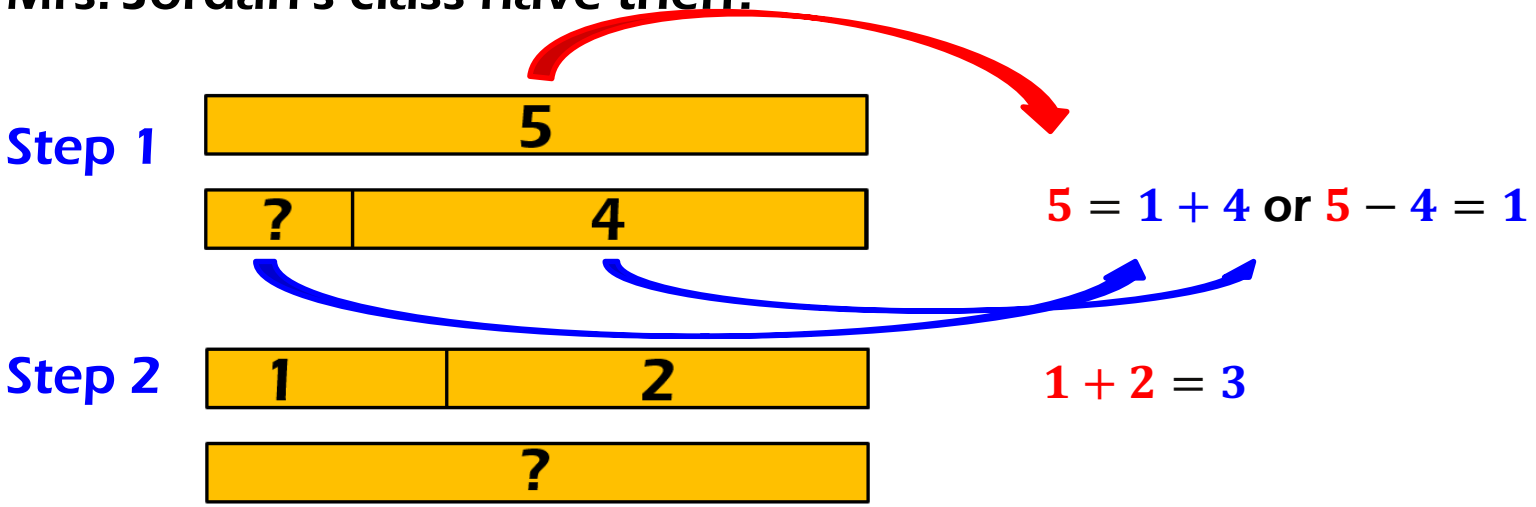


1. Draw an equal sign
2. On one side, draw the values represented on one line.
3. On the other side, draw the values represented by the other line.

They are the same length!

Notice the directions do not specify which line to put where.

$$5 = 1 + 4$$

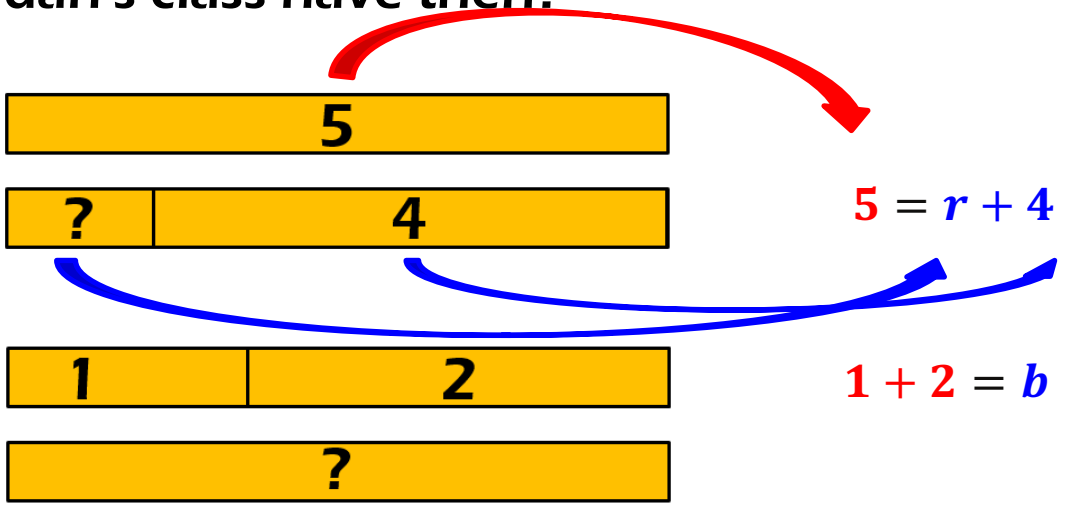
is just as valid as

$$1 + 4 = 5.$$

This helps in seeing the equal sign as equivalence rather than an operator. I have done it both ways above.

Connecting to the Abstract Stage with a Variable

Mrs. Jordan's class borrowed 5 books from the library. They returned 4 books. Later, the class borrowed 2 new books. How many books did Mrs. Jordan's class have then?



$5 = r + 4$ or $5 - 4 = r$

$1 + 2 = b$

1. Draw an equal sign
2. On one side, draw the values represented on one line.
3. On the other side, draw the values represented by the other line.

They are the same length!

Could I or should I have used the variable b for both?

r = remaining # of books after return
b = amount after borrowing books

Think about it & discuss it with your partner.

Final Level of Abstraction: Full Symbolic

Mrs. Jordan's class borrowed 5 books from the library. They returned 4 books. Later, the class borrowed 2 new books. How many books did Mrs. Jordan's class have then?

Step 1 $5 - 4 = 1$

Step 2 $1 + 2 = 3$

Avoid

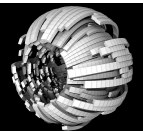
$$5 - 4 = 1 + 2 = 3$$

as

$$5 - 4 \neq 1 + 2$$

and

$$5 - 4 \neq 3$$



Second Grade: Two Step Problems with CRA

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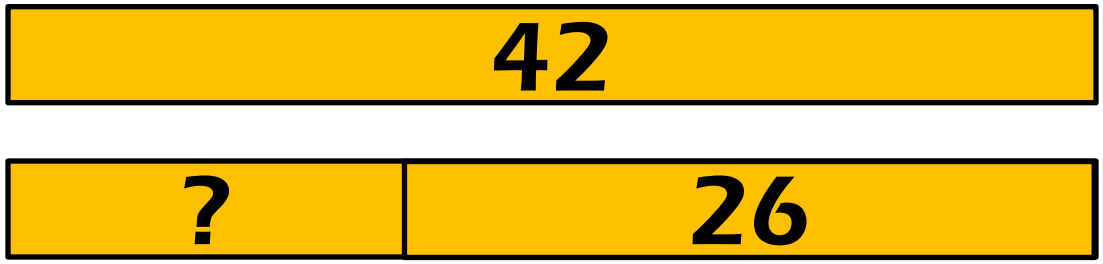
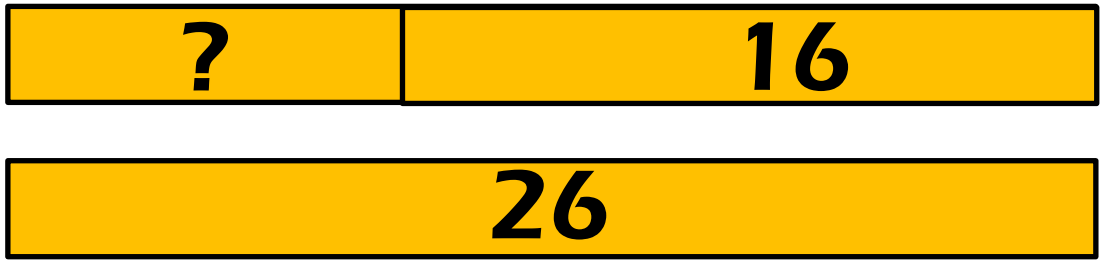
Representational Stage (not to scale)

Astro works 42 hours at both the Black Hole and the Galaxy Grill. He works 26 hours at the Black Hole. How many fewer hours does Astro work at the Galaxy Grill?

Step 1



Step 2



Third Grade: Two Problems with CRA, Alexa & Max

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Secondary Math TOSA
Hemet Unified
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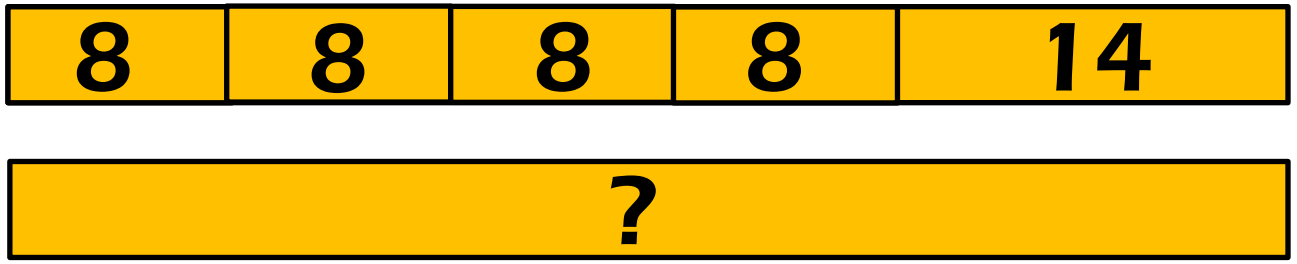
Representational Stage (scale & not to scale)

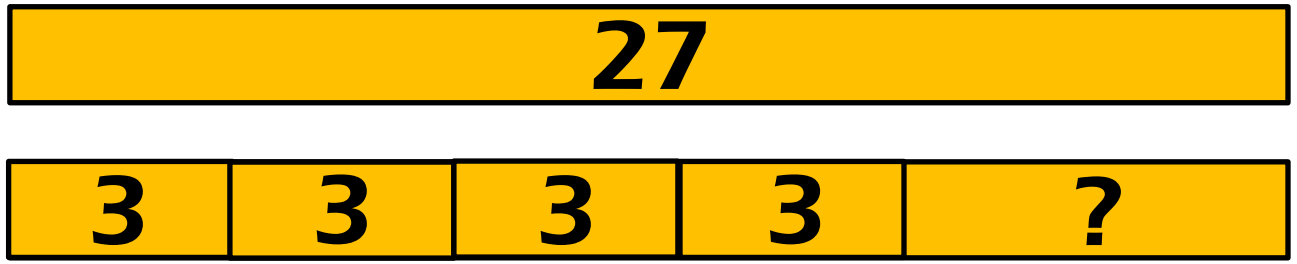
Max read 8 pages of his detective book every day. After reading for 4 days, Max still had 14 pages left. How many pages are in Max's detective book?

Step 1



Alexa had 27 erasers. She gave 3 erasers to each of her 4 friends. How many erasers did Alexa have left?

Step 1



Fourth Grade: Comparing Problem with CRA

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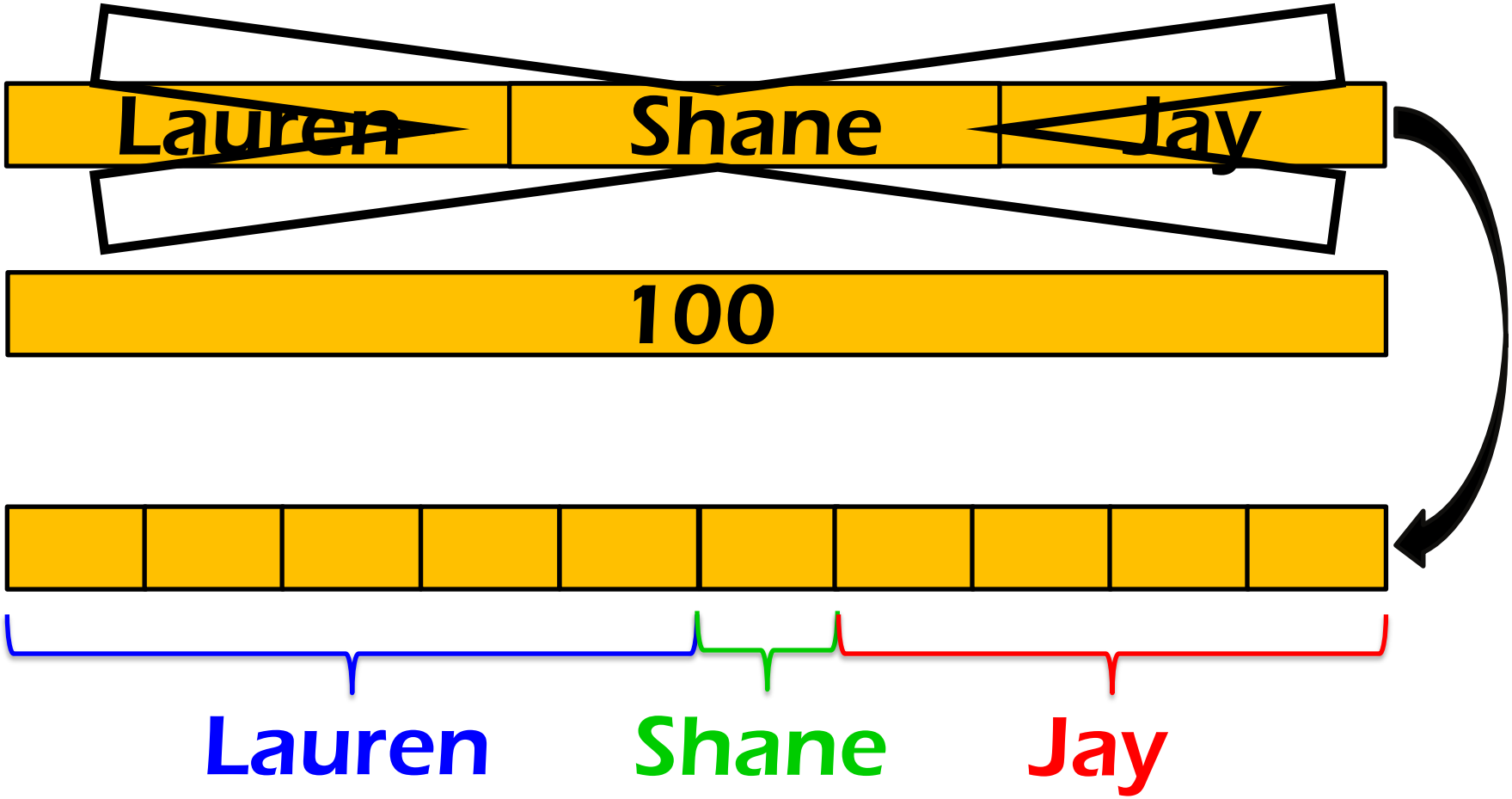
Representational Stage (to scale)

Lauren, Shane & Jay shared 100 crayons. Lauren received 5 times as many crayons as Shane. Jay received 4 times as many crayons as Shane. How many more crayons did Lauren receive than Shane?



Fifth Grade:
Fraction Problem with CRA

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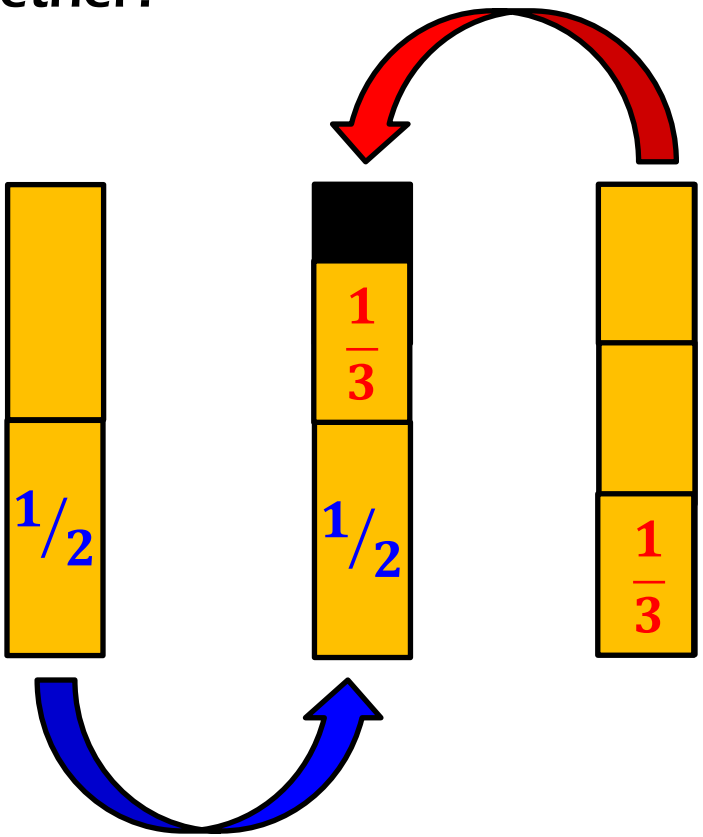
Representational Stage (Drawing)

Bob the Baker needs 1/2 cup of milk to make triple fudge cookies and 1/3 cup of milk to make sugar cookies. How much milk does Bob the Baker need altogether?



What do you notice?

What do you wonder?

Annie Fetter

https://www.youtube.com/watch?v=a-Fth6sOaRA

Sixth Grade: Dividing Fractions Problem with CRA

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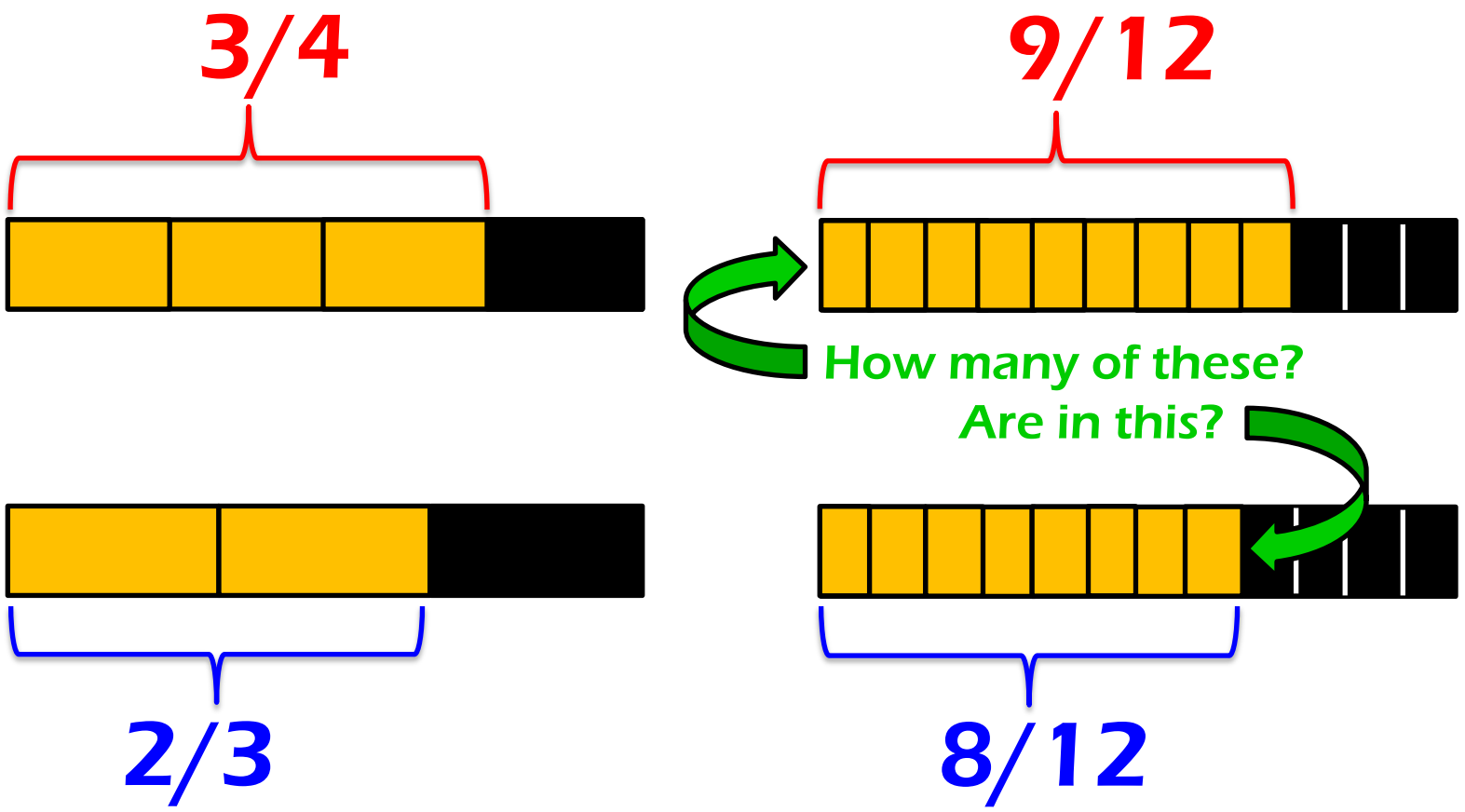
Representational Stage (Drawing)

How many 3/4 cup servings are in 2/3 of a cup of yogurt?



Cut the fourths into thirds.

Cut the thirds into fourths.

To find the common denominator of twelfths.

Seventh Grade: Proportion & Percent Problems with CRA

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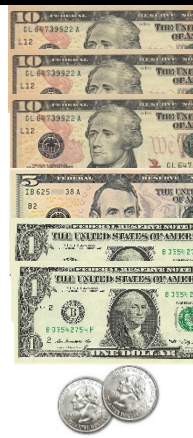
Connecting to the Abstract Stage

A sweater is marked down 30%. The original price was $37.50. What is the price of the sweater after it is marked down?

Consider a double bar graph for proportionality.



Percent		Value	
100%	=	$37.50	
30%	70%	Discount	New Price

You can write the proportion just like it looks on the tape diagram.

Eighth Grade: Solving Linear Systems with CRA

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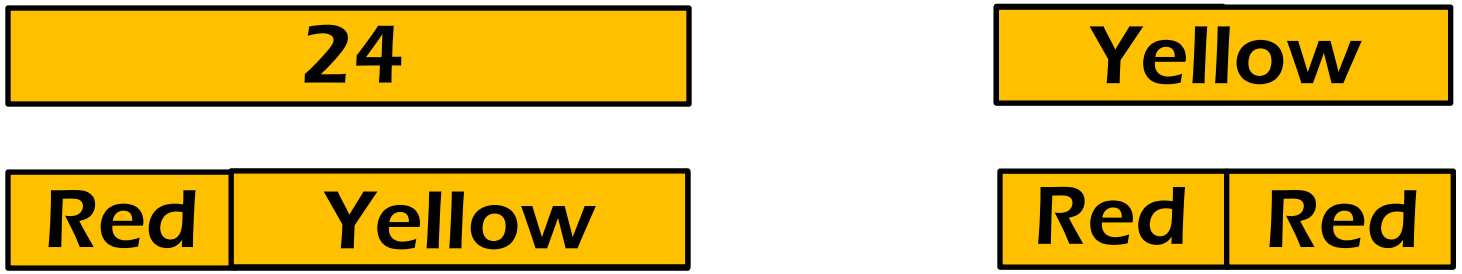
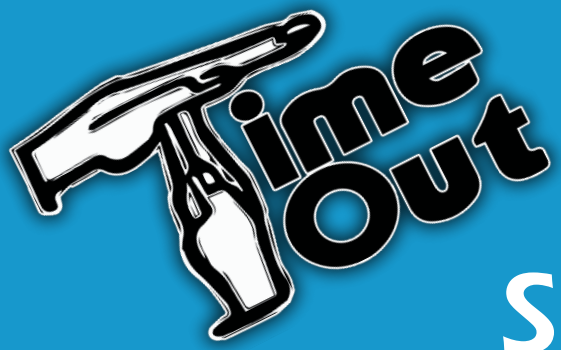
Representational Stage (to scale)

Each year, Quinn plants 24 flowers in his garden. This year, he planted only red flowers and yellow flowers. Quinn prefers yellow, so he planted twice as many yellow flowers as red flowers.



They could draw flowers (circles) to solve the problem; however, tape diagrams can help students understand solving linear systems by substitution.



Scenic Overlook: NCTM's Effective Math Teaching Practices

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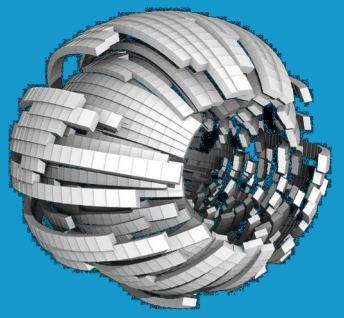

Which Effective Mathematics Teaching Practices are in Play?

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Effective Mathematics Teaching Practices

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Scenic Overlook: Try One

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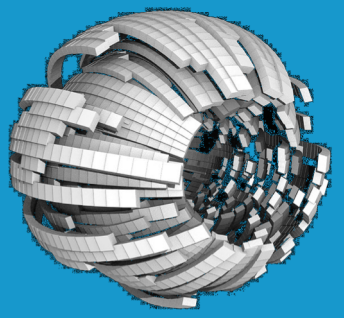
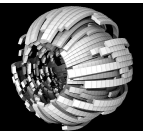


Try One!

1. Choose one:
   a. The Challenge Problem
   b. A Problem from your Text
   c. A Problem from www.ThinkingBlocks.com
2. Work it out alone or with a partner
3. Please complete our survey at …



Want to Learn More?

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Challenge Problem: An Argument for Tape Diagrams

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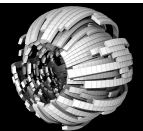
"Challenge" Problem

Nicole had 7 times as many nickels as Lexi. After Nicole gave Lexi some of her nickels, each girl had 20. How many nickels had Nicole given to Lexi?



Handouts

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National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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www.nctm.org/principlestoactions

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