Connections for Math Learning & Retention

The Role of Context, Conceptual Understanding, and Concreteness Fading in Developing Student Understanding and Procedural Fluency

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Procedural Fluency

How would you define it, and how is it built ?

Reflective Writing

How is Procedural Fluency Built ?

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There is no Spoon!

There is no Spoon! Procedures in Isolation do not Work

We want students to be procedurally fluent
so they can be the doers of math;
therefore, we teach procedures
to the point of spoon feeding.
We try to bend the spoon.

Do not try and bend the spoon. That's impossible.

It is not the spoon that bends,
it is only yourself…

The truth is
students need to be doers of math
to be procedurally fluent.
The answer does not lie outside the student with the procedures,
but inside the student with the connections they make between the conceptual mathematics.
We need to bend the person.

Effective Mathematics Teaching Practices

#3: Use and connect mathematical representations.
"Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving." Principles to Actions, page 10

#6: Build procedural fluency from conceptual understanding.
"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems."
Principles to Actions, page 10

#8: Elicit and use evidence of student thinking.
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.
Principles to Actions, page 10

“The Illusion of Mastery”

Some may succeed temporarily by utilizing innate procedural memory, they bend the spoon with their brute strength; however, others are not strong enough, that is, they do not possess the same amount of innate procedural memory.

The problem is that even those that can do the work by memorizing lengthy procedures often do not know when it applies or cannot apply it to a problem that is only slightly different; they do not know when to bend the spoon or how to bend a fork.

Still others simply forget due to the lack of connections being made, and/or the lack of interleaved practice. They used to be able to bend the spoon, but they forgot how.

Consider the following student, who learned the procedure…

Our Why: One of Our Students in Our District

Does it sound like the student knows how to do it from this open-ended response?

Extra Credit +4

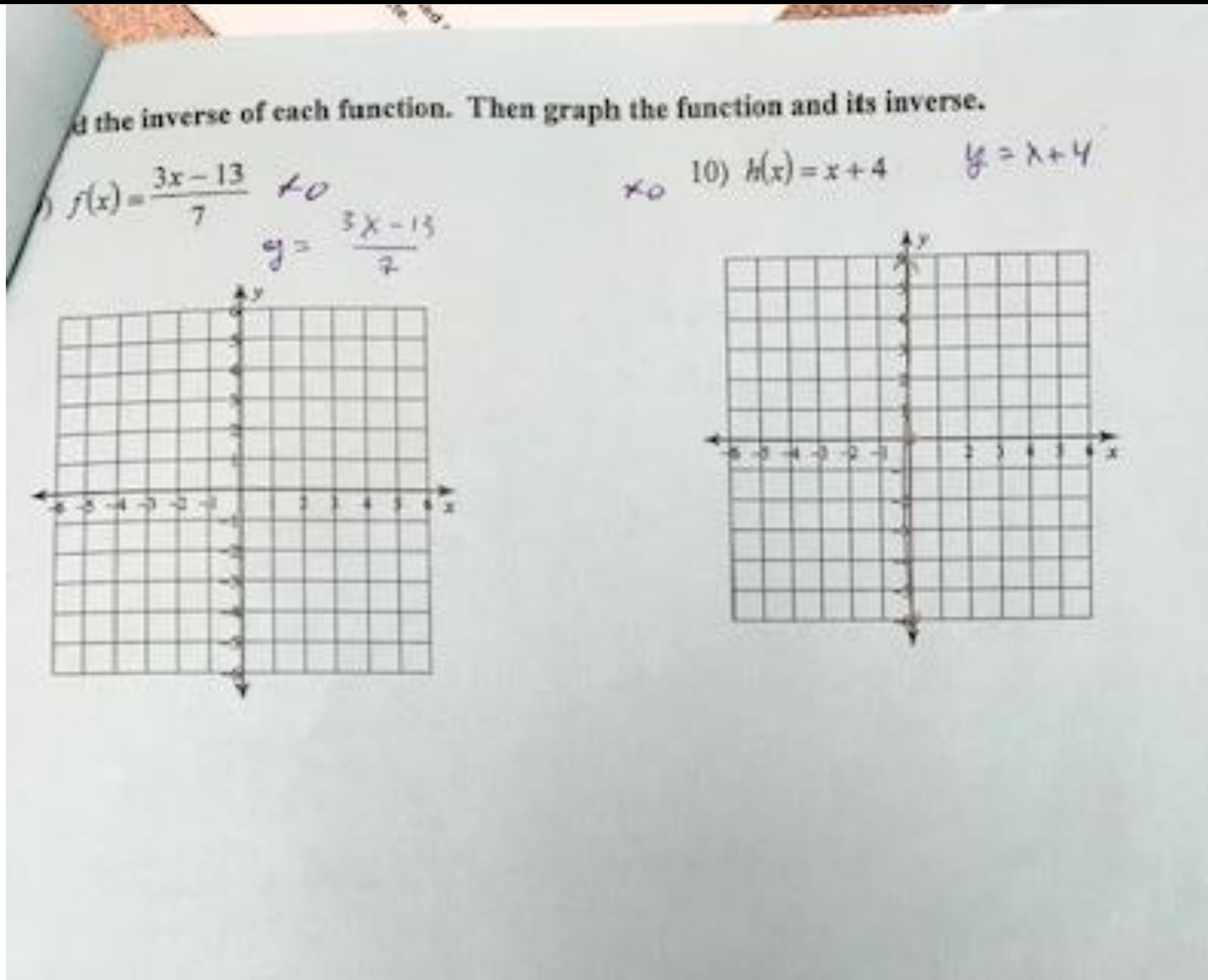
Explain how to find the inverse of a function and what happens to the domain and range of the inverse function.

You replace $f(x)$ with y then switch the x's and y's and then you solve for y. The domain becomes the range and the range becomes the domain. They switch.

Our Why: One of Our Students in Our District

This problem did not even ask for application to a real life scenario. It was "naked math," a straight application of the procedure the student was able to memorize and explain.

The student did not even know it applied here. He did not even know it was the spoon.



Procedural Fluency

How would you define it, and how is it built ?

Add to your writing, pair,

share any additions you made

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Agenda

- ✓ There is No Spoon: How is Procedural Fluency Built?
- ❑ Scaffolding: What It Is and Is Not
- ❑ Lesson Illustrating:
  - ❑ Elicit and use evidence of student thinking
  - ❑ Use and connect mathematical representations
  - ❑ Build procedural fluency from conceptual understanding
- ❑ Takeaways

Scaffolding

How would you define it, and what would you look for as evidence of it in the classroom?

Reflective Writing

Scaffolding vs. Spoon-Feeding

What Scaffolding Is and Is Not

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Scaffolding as Spoon-feeding

"I am scaffolding to help my struggling students, and by that I mean: I am breaking down the lesson into bite size chunks, making sure students understand each one before moving on."

"Oh, so you mean you are removing my productive struggle?"

Scaffolding as Spoon-feeding

"I am scaffolding to help my struggling students, and by that I mean: I am breaking down the lesson into bite size chunks, making sure students understand each one before moving on."

"Oh, so you mean that you are the doer of mathematics as opposed to me being the doer of mathematics?"

Scaffolding as Spoon-feeding

"I am scaffolding to help my struggling students, and by that I mean: I am breaking down the lesson into bite size chunks, making sure students understand each one before moving on."

"Oh, so you mean I cannot engage in higher ordered thinking without knowledge of basic skills? When will I be given the time to do higher ordered thinking in your system?"

Scaffolding as Spoon-feeding

"I am scaffolding to help my struggling students, and by that I mean: I am breaking down the lesson into bite size chunks, making sure students understand each one before moving on."

"Oh, so you mean you are doing what is easy for the teacher, breaking topics down into bite-size pieces with little consideration for my thinking?"

Scaffolding as Spoon-feeding

"I am scaffolding to help my struggling students, and by that I mean: I am breaking down the lesson into bite size chunks, making sure students understand each one before moving on."

"Oh, so you mean I don't have access to grade level content as you go back to review prior levels?"

Scaffolding as Spoon-feeding

Spoon-feeding is not only
not beneficial, it is not scaffolding…

Why is the Achievement Gap Widening?

- GRAPH HERE?

Why is the Achievement Gap Widening?

- I have to "scaffold" equations
  1. One step (6^{th} Grade)
  2. Two step (7^{th} Grade)
  3. Distributive
  4. Variables on both sides
  5. Multistep Equations
- Ninth grade algebra starting with 6^{th} grade, one step equations, then 7^{th} grade, etc.
- Worse when compounded by extended beginning of the year review

- Why am I out of time?
- "Those students" get less access to grade level material and the achievement gap widens, because of who?
- Even if students taught procedurally remember for a procedural unit test, what happens when the prompts are not as procedural or the following year when students have forgotten?

Scaffolding Defined

Instructional scaffolding is the support given during the learning process… to promote a deeper level of learning… These supports may include the following:

- **resources [using pictorial math provides a resource to employ]**
- **a compelling task [not a simplified task]**
- **templates and guides [column notes for multiple representations]**
- **guidance on the development of cognitive skills**
  [How are students thinking about what they are doing?]

These supports are gradually removed as students develop autonomous learning strategies, thus promoting their own cognitive, affective and psychomotor learning skills and knowledge. Teachers help the students master a task or a concept by providing support [not simplifying content]. The support can take many forms such as outlines, recommended documents, storyboards, or key questions.

https://en.wikipedia.org/wiki/Instructional_scaffolding

Scaffolding

How would you define it, and what would you look for as evidence of it in the classroom?

Add to your writing, pair,

share any additions you made

THERE IS NO SPOON

Lesson

Elicit and use evidence of student thinking

Use and connect mathematical representations

Build procedural fluency from conceptual understanding

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Student View

I went through the drive-thru and gave them $10.

I got back two hamburgers and $4.

How much did the burgers cost?

Context as a Scaffold - Equations

I went through the drive-thru and gave them $10. I got back two hamburgers and $4. How much did the burgers cost?

What did you do to solve this?

If h= the cost of the burgers, then:

$$2h + 4 = 10$$

$$-4 = -4$$

Why did you do this (consider the context)?

$$2h = 6$$

$$\frac{2}{2}h = \frac{6}{2}$$

Why did you do this (consider the context)?

$$h = 3$$

Context as a Scaffold - Equations

I went through the drive-thru and gave them $10. I got back two hamburgers and $4. How much did the burgers cost?

Students can do this problem without instruction in how to solve an equation, because the context is the scaffold. This refutes the argument that students must start with one step equations before solving two step equations.

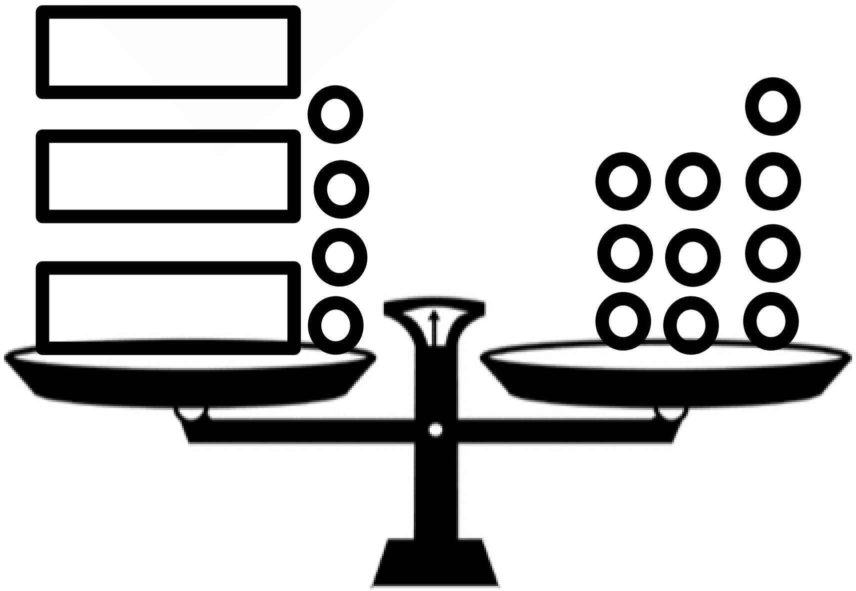
Start with the conceptual, see how students are thinking about it, and then formalize a procedure based on what they did.

Linear Equations: How Many Peas in the Pod?

Another Context for Linear Equations

- How are the peas arranged in a pod?
- You can think of a pea pod as a linear term.
- Let represent the peapod.
- Let represent the peas

If p= # of peas in a pod, then…



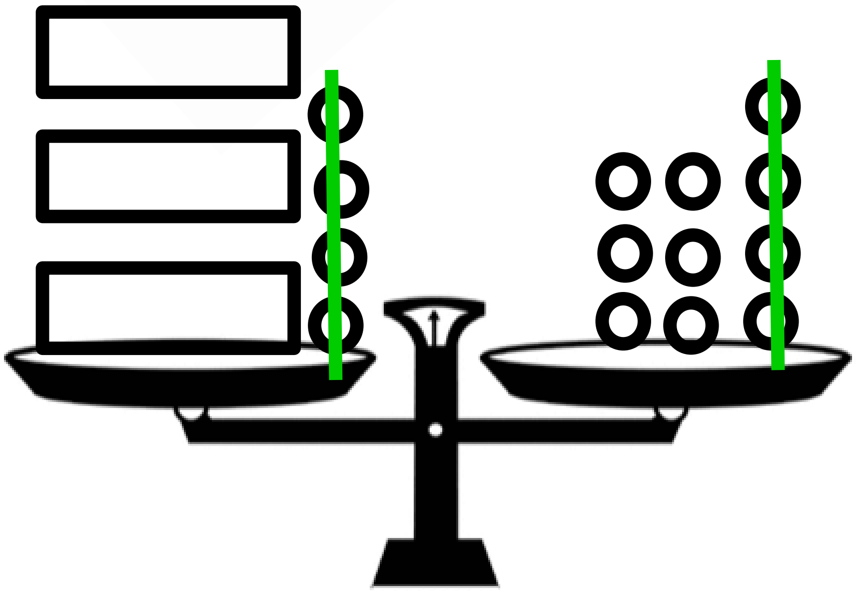
If p= # of peas in a pod, then..

How many peas in the each pea pod? (Disregard the weight of the pod.)

How did you do it?

$3p + 4 = 10$

Context for Equations



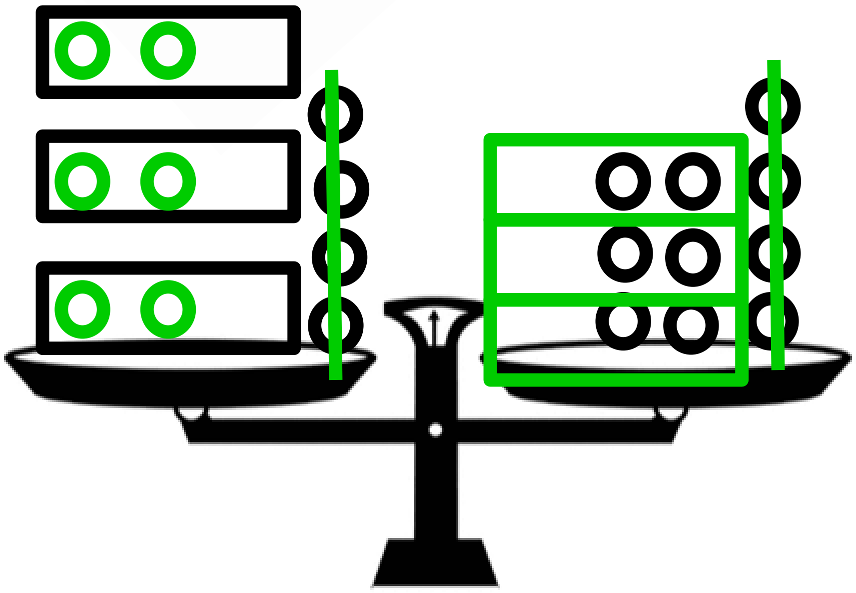
If p= # of peas in a pod, then..

How many peas in the each pea pod? (Disregard the weight of the pod.)

How did you do it?

$$3p + 4 = 10$$

$$3p + 4 - 4 = 10 - 4$$



If p= # of peas in a pod, then..

How many peas in the each pea pod? (Disregard the weight of the pod.)

How did you do it?

$$3p + 4 = 10$$

$$3p + 4 - 4 = 10 - 4$$

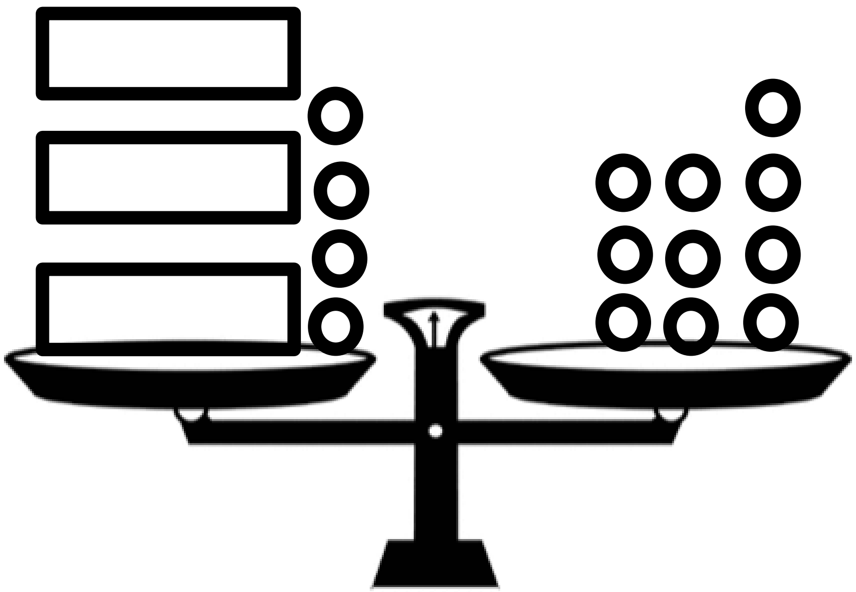
$$3p = 6$$

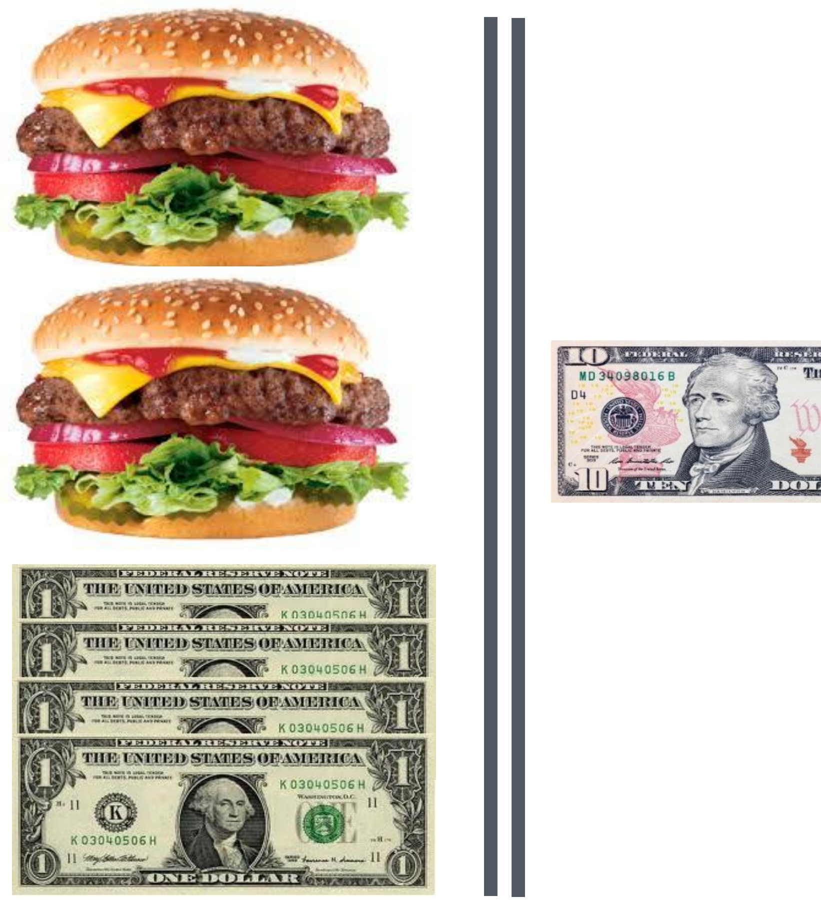
$$\frac{3}{3}p = 6/3$$

$$p = 2$$

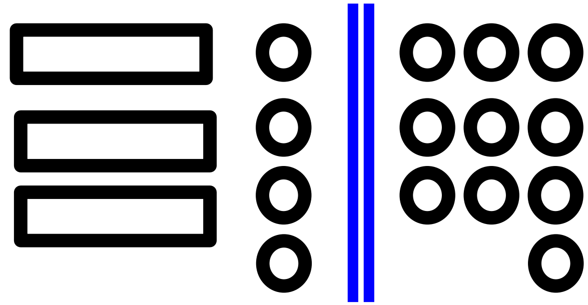
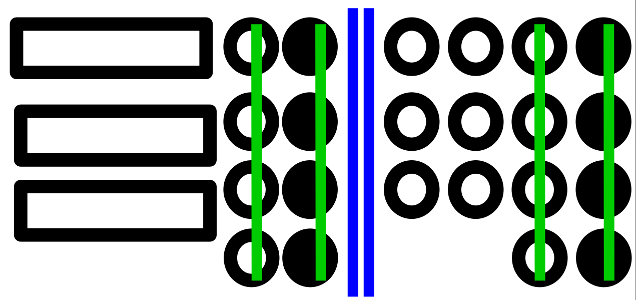
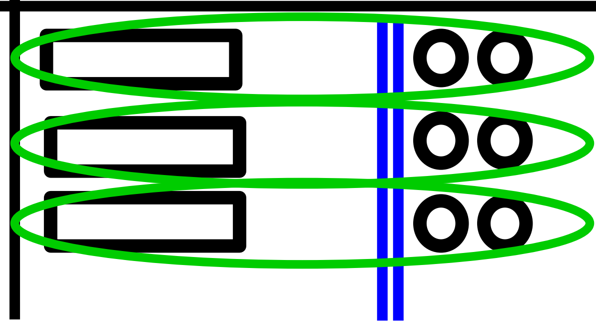
Time for a Reflective Question

How is the problem with the peas **similar** to the hamburger problem?





Linear Equations: Four Column Notes

Symbolic	Pictorial	What You Did	Property
$3p + 4 = 10$		Drew the Pictures	Given
$3p + 4 = 10$ $-4 = -4$		Subtracting four from both sides is the same as adding a negative four to both sides.	Addition Property of Equality
$3p = 6$ $3/3p = 6/3$ $p = 2$		I divided both sides by three.	Division Property of Equality

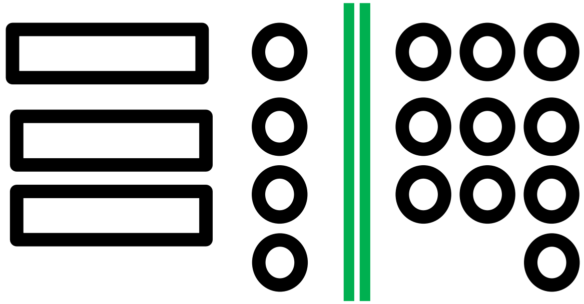
Try One with Algebra Tiles & Your Focus Notes

Symbolic	Pictorial	What You Did	Property
$4p + 3 = 7$			

Linear Equations: Four Column Worksheet

Symbolic	Pictorial	What You Did	Property
$3p + 4 = 10$			

Linear Equations: Four Column Worksheet

Symbolic	Pictorial	What You Did	Property
			

Linear Inequalities without Spoon-feeding

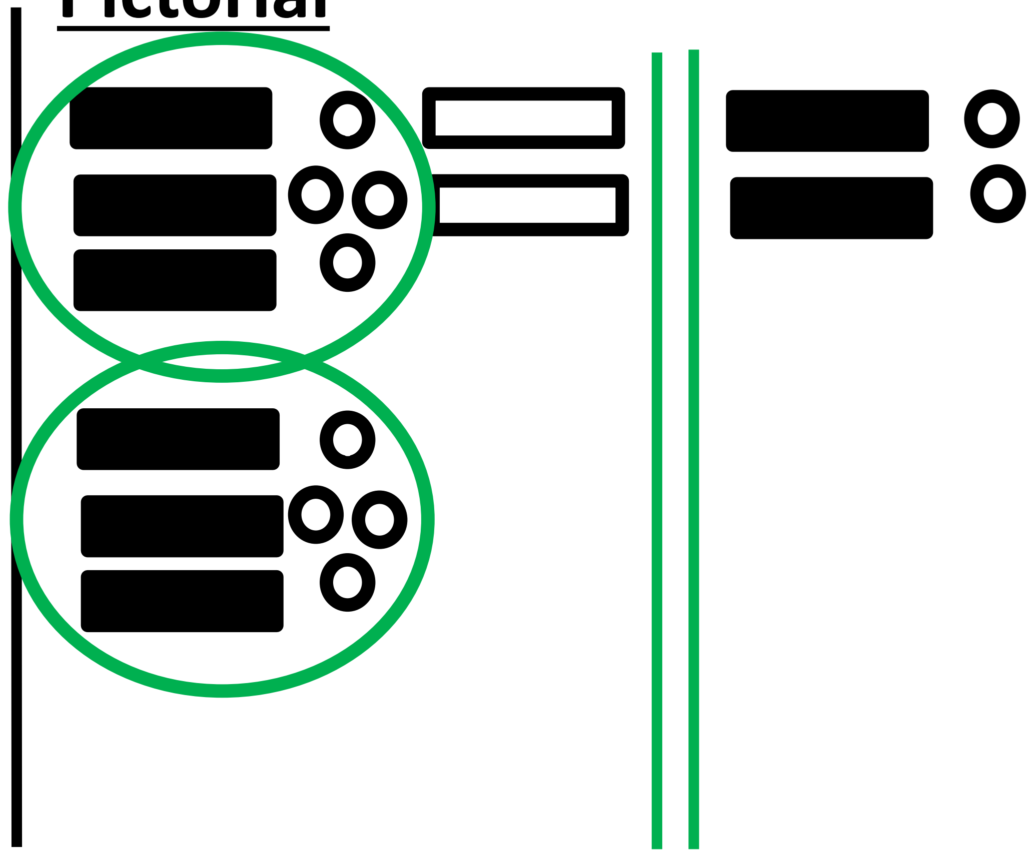
Symbolic	Pictorial	What You Did	Property

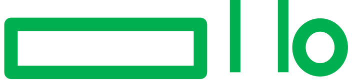
I don't have the space here to show all four columns so I will do the symbolic and pictorial only.

Inequalities: Given - Draw the Shapes, Draw the Goal

Symbolic | **Pictorial**

$2(-3x + 4) + 2x < -2x + 2$





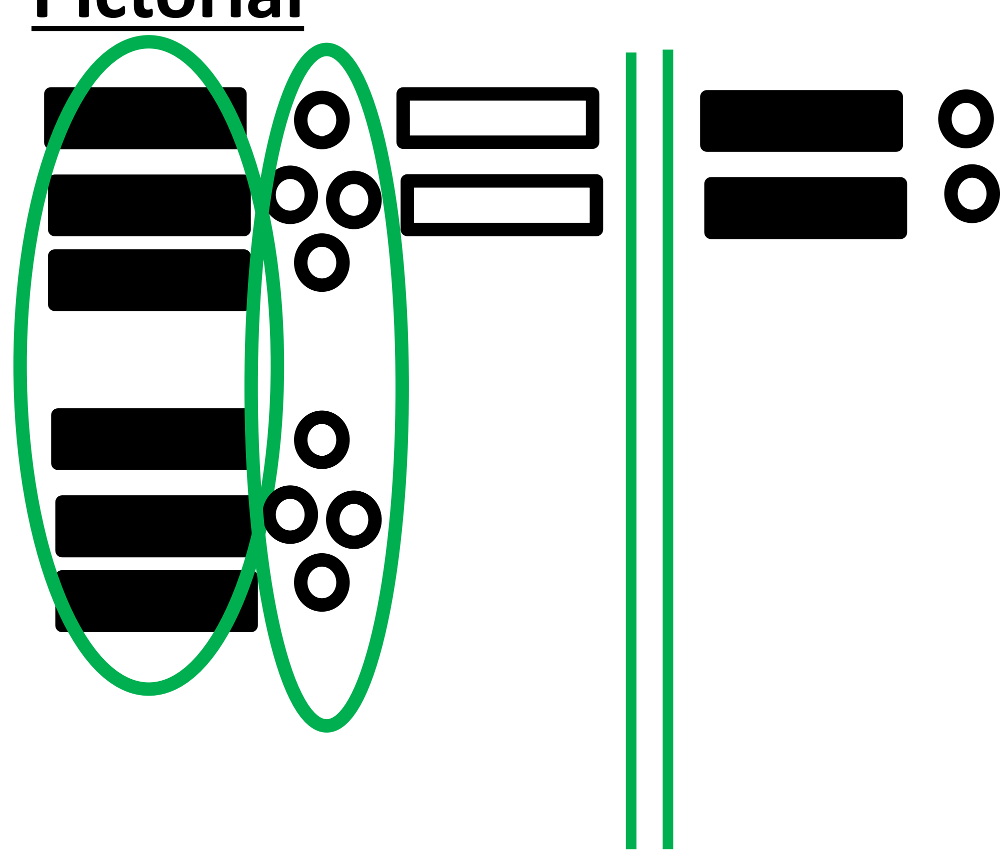
Students usually do not know where to begin so I have them draw the shapes and draw the goal as the first step.

Inequalities: Given – Distributive Property

Symbolic

$2(-3x + 4) + 2x < -2x + 2$

Pictorial



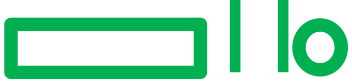
Reading the shapes horizontally is what is given.
Reading the shapes vertically is the same as distributing.

Inequalities: Given – Distributive Property

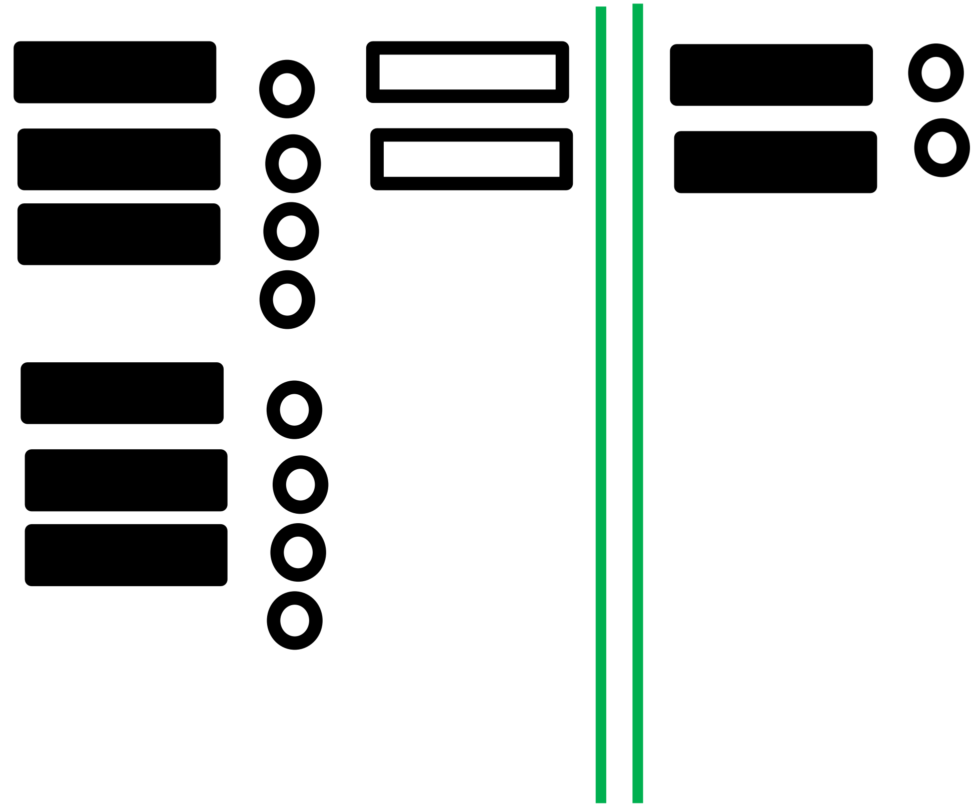
Symbolic

$2(-3x + 4) + 2x < -2x + 2$

$-6x + 8 + 2x < -2x + 2$



Pictorial



Emphasize that you have to distribute to even draw the shapes.

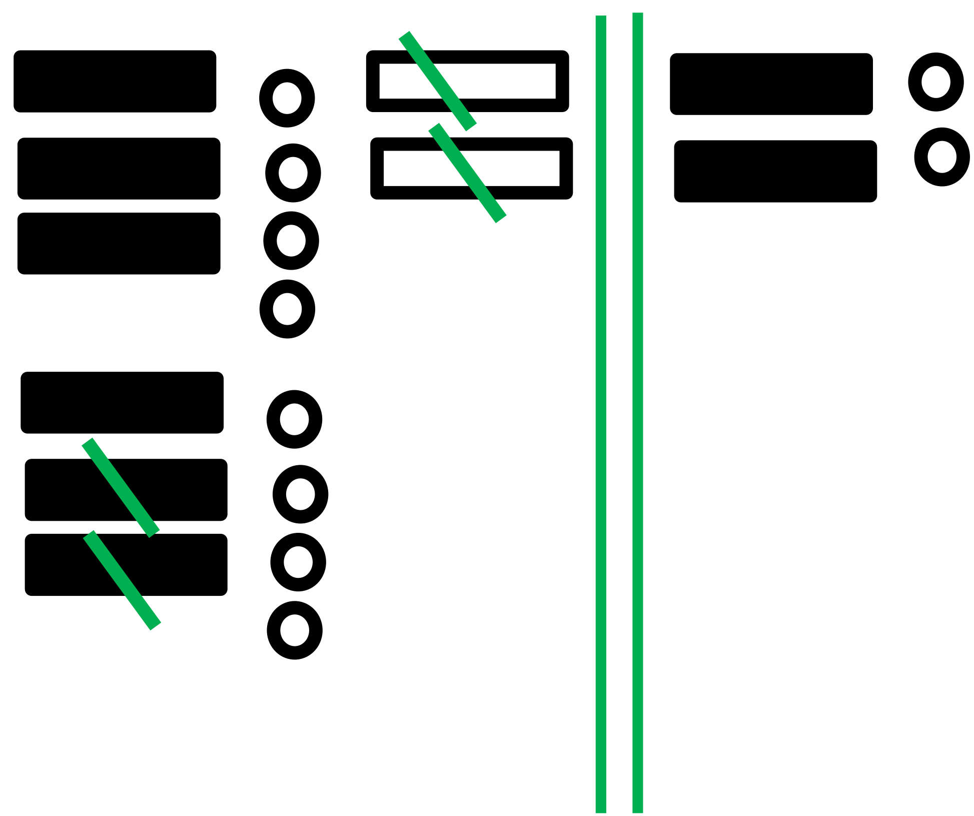
Inequalities: Combine Like Terms & Additive Inverse

Symbolic | **Pictorial**

$2(-3x+4)+2x < -2x+2$

$-6x+8+2x < -2x+2$

$-4x+8 < -2x+2$

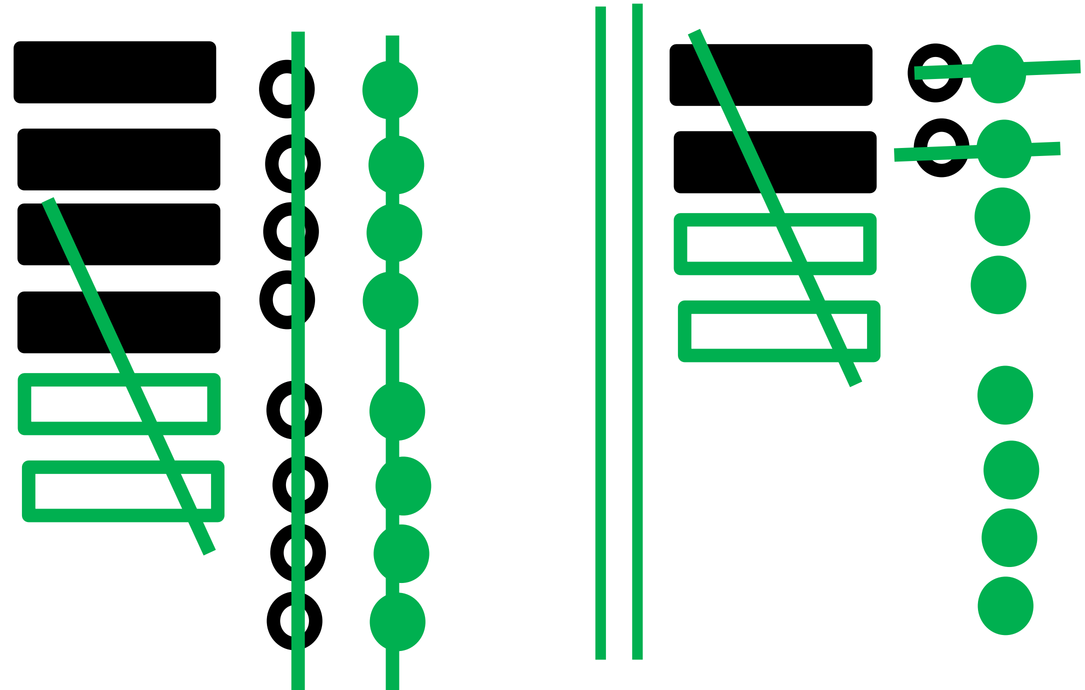


Equations are just two equivalent expressions. Simplify each side.

Inequalities: Addition Property of Equality

Symbolic	
$2(-3x + 4) + 2x <$	$-2x + 2$
$-6x + 8 + 2x <$	$-2x + 2$
$-4x + 8 <$	$-2x + 2$
$+2x - 8$	$+2x - 8$

Pictorial



What makes a zero pair?

Variables on one side & constants on the other.

You can do this in two steps. Once they understand the goal though, they will be comfortable with the "double switch."

Inequalities: Multiplication Property of Equality

Symbolic

$2(-3x + 4) + 2x < -2x + 2$

$-6x + 8 + 2x < -2x + 2$

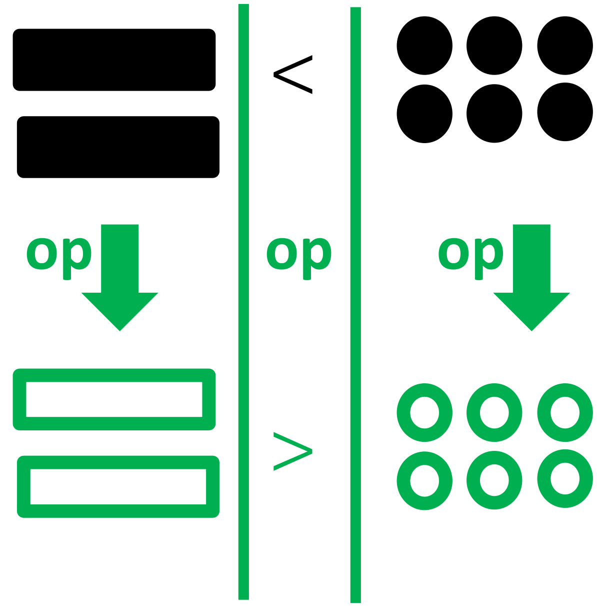
$-4x + 8 < -2x + 2$

$\underline{+2x - 8} \qquad \underline{+2x - 8}$

$-2x(-1) < -6(-1)$

op op op

Pictorial



My goal is one positive unknown, x. I need the opposite of what I have so multiply both sides by -1. I have students write op-op-op, especially for knowing when to switch the signs in inequalities. Write it three times; do it three times. Notice this gets rid of any double negative division.

Inequalities: Division Property of Equality & Solution

Symbolic	Pictorial

$2(-3x+4)+2x < -2x+2$

$-6x+8+2x < -2x+2$

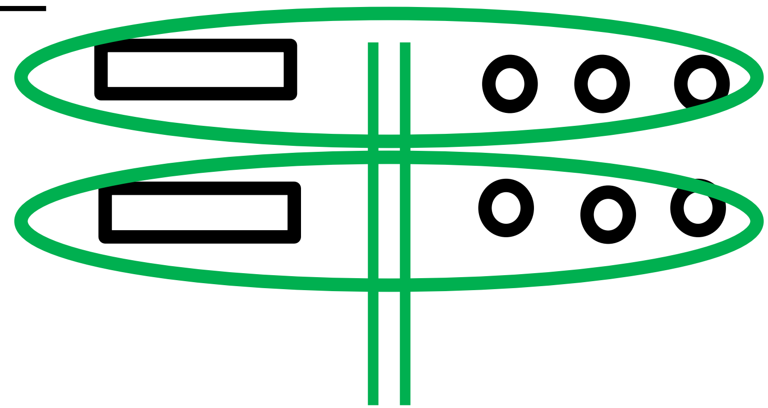
$-4x+8 < -2x+2$

$+2x-8 \qquad +2x-8$

$-2x(-1) < -6(-1)$

$\dfrac{2x}{2} > \dfrac{+6}{2}$

$x > 3$



Make the coefficient one. If 2x contain 6, then how many in one?

Did I subtract? What operation did I use?

Divide by the coefficient to make a single x.

Next Level of Abstraction
Drawing Around the Symbols

$5 + 2(-3x + 4) + 2x \geq -2x + 1$

$+5 \quad \boxed{-6x} \quad +8 \quad \boxed{+2x} \quad \geq \quad \boxed{-2x} \quad +1$

$\boxed{-4x} \quad +13 \quad \geq \quad \boxed{-2x} \quad +1$

$\boxed{+2x} \quad -13 \qquad +2x \quad -13$

Same team or do they fight?

$-2x(-1) \geq -12(-1)$

op op op

$\frac{2x}{2} \leq \frac{+12}{2}$

$\boxed{x} \leq 6$

This is a good time to introduce larger coefficients or even rational coefficients.

Where do I begin?

You cannot draw shapes around parenthesis so distribute first to get rid of them.

Now draw your shapes & your goal.

Put variables on the left and circles on the right.

Opposite Side, Opposite Sign.

WAIT! Op-Op-Op (Multiply by Negative One)

Divide by the coefficient.

Final Level of Abstraction
Just Do It

$5 + 2(-3x + 4) + 2x \geq -2x + 1$

$+5 \quad -6x \quad +8 \quad +2x \quad \geq -2x \quad +1$

$-4x \quad +13 \quad \geq \quad -2x \quad +1$

$+2x \quad -13 \qquad\qquad +2x \; -13$

$-2x(-1) \geq -12(-1)$

op op op

$\frac{2x}{2} \leq \frac{+12}{2}$

$x \leq 6$

Where do I begin?

You cannot draw shapes around parenthesis so distribute first to get rid of them.

Now draw your shapes & your goal.

Put variables on the left and circles on the right.

Opposite Side, Opposite Sign.

WAIT! Op-Op-Op (Multiply by Negative One)

Divide by the coefficient.

Giving Students the Procedures?

Students should generalize to procedures themselves so they are more likely to remember them.

If you must provide them, then they should be a temporary resource for students. Keep them simple, short and always true:

1. Draw the shapes and the goal [this will help them begin]
2. If there are parenthesis, then you don't have a single group so you cannot draw the shapes. You must distribute first, if necessary. 3(2x +1) is not equivalent to 32x + 1. NOT $\boxed{3(2x}$ +1
3. Simplify each equivalent expression, if possible.
4. Collect variables on one side & constants on the other.
5. Make the coefficient of the variable one.

Standards of Mathematical Practice

Make sense of problem and persevere in solving them.	Attend to precision.
Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others.
Model with mathematics.	Use appropriate tools strategically.
Look for and make use of structure.	8. Look for and express regularity in repeated reasoning.

At what point in the lesson did these occur?

Modeling happened when we used symbols & equations to find the cost of the burger.

The use of manipulatives falls under tools not modeling.

Takeaways

What Does This Have to Do with Leading Instruction?

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Possible Questions to Pose Teachers

How could you change this lesson to engage students in some productive struggle?

How could you present this lesson so students are the doers of mathematics as opposed to you?

How can you engage students in higher ordered thinking regardless of their knowledge of basic skills?

How could you elicit and build on student thinking in this lesson?

How could context support students in this lesson rather than going back to reteach prior grade levels?

How could procedures be teased from conceptual understanding so students make connections and might be more likely to remember the procedures used today?

What other ways are there to represent this concept and what connections can you help students make between them?

How could you start with something concrete then utilize drawings and diagrams before moving into the abstract symbolic representation?

I saw some work on procedural fluency today. How have you/could you help students build that fluency through conceptual understanding?

How could inquiry or context engage students in the standards for mathematical practice?

I noticed you provided the students with the procedure today; are there any ways students could have generalized the procedure from what they did?

Handouts

Key Information from

Kyle Pierce on Concreteness Fading and

Principles to Actions

Ensuring Mathematical Success for All

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Levels of Abstraction = Concreteness Fading
Concrete, Representational, Abstract

During the first half of my teaching career, I would spend what seemed to be the first half of a math lesson teaching a new math concept by **sharing definitions, formulas, steps and procedures.**

To make things more challenging for my students, **I would simultaneously introduce the symbolic notation** used to represent those ideas. Then, I would spend the remainder of the lesson **attempting** to help my students make sense of these very **new** and often **abstract ideas**.

By the end of the lesson, I could help many students build an understanding, but **there was always a group I felt who I would leave behind.** Like many other teachers, I was just **teaching in a very similar way to that how I was taught**. **I knew no different.**

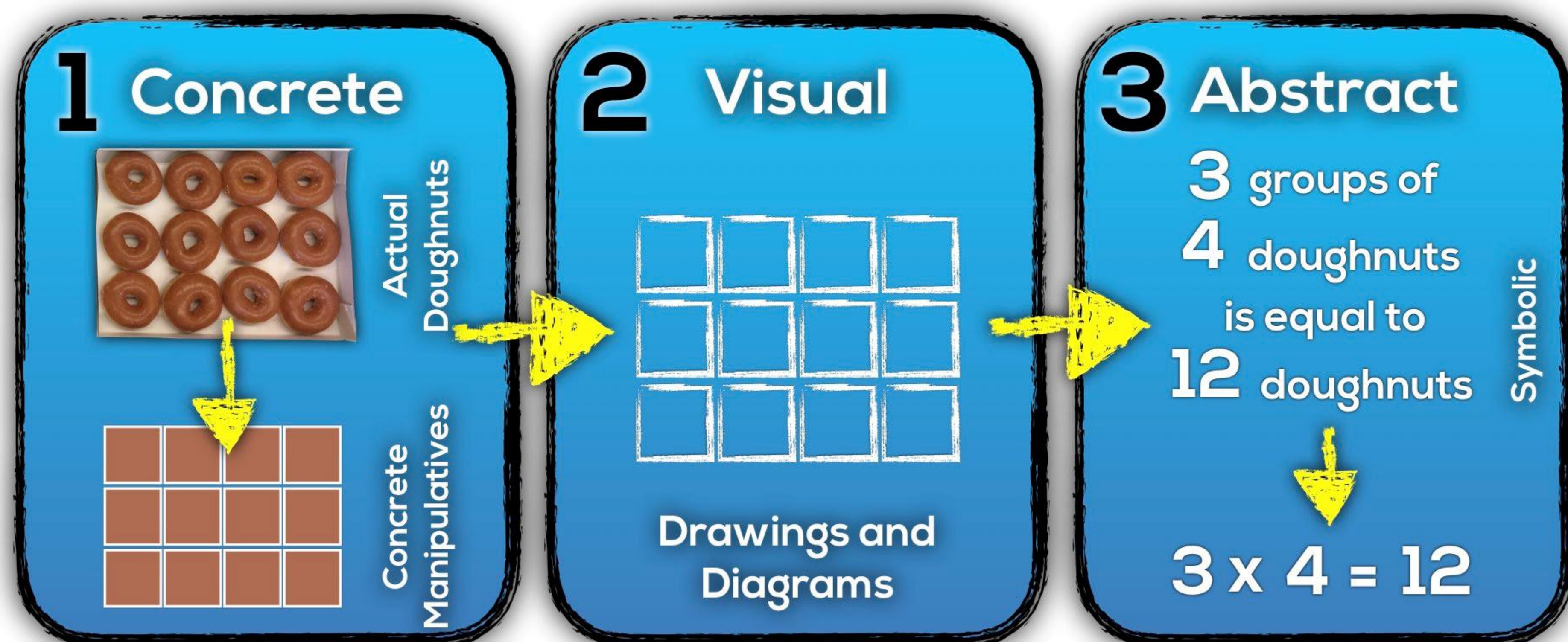
However, if we consider that **new learning requires the linking of new information with information they already know and understand**, we should be intentionally planning our lessons with this in mind. A great place to start new learning is through the use of a **meaningful context** and **utilizing concrete manipulatives** that students can touch and feel.

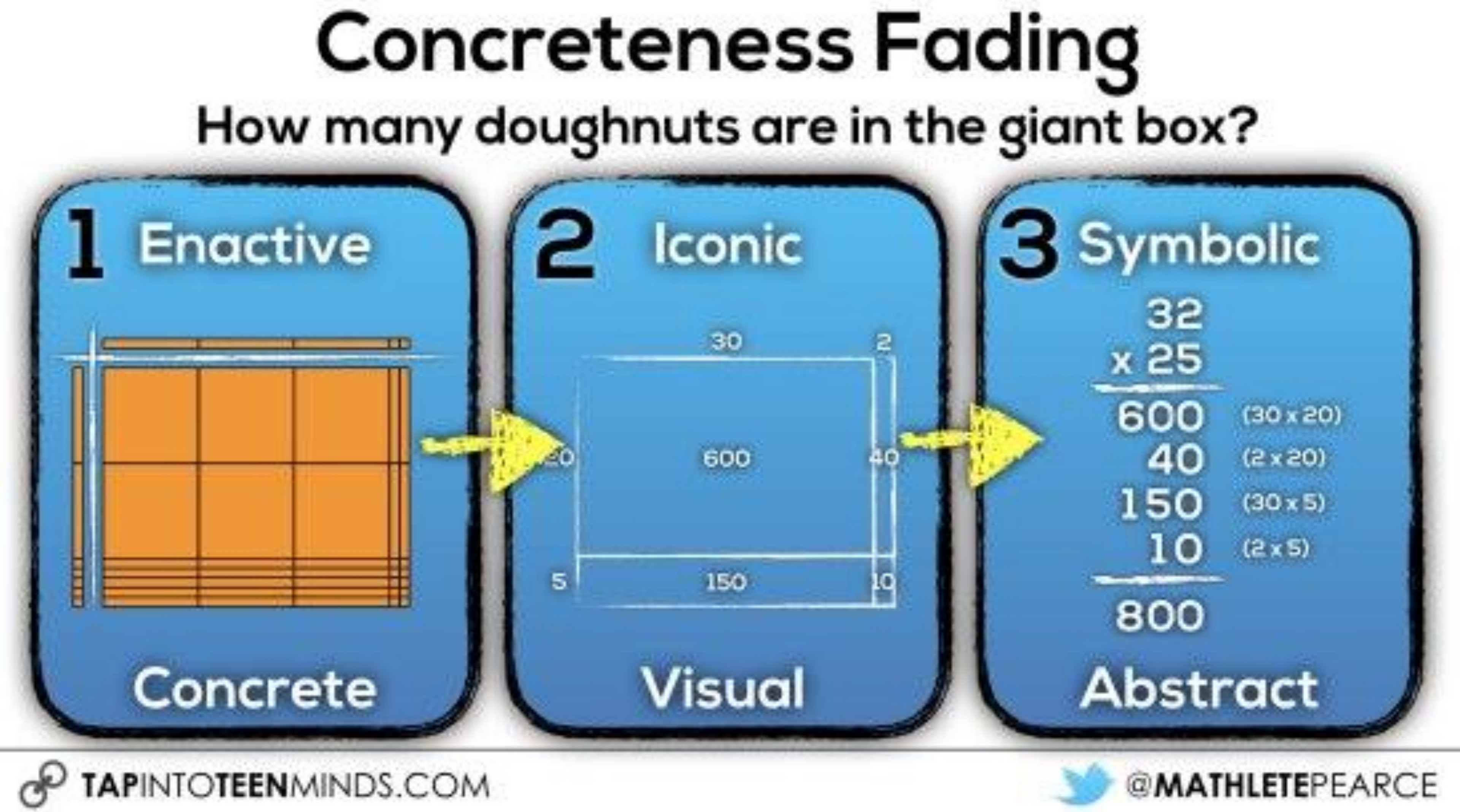
When we teach in this way, we **minimize the level of abstraction** so students can **focus their working memory on the new idea** being introduced in a meaningful way.

- Kyle Pierce, Tap into Teen Minds, https://tapintoteenminds.com/concreteness-fading/

Concreteness Fading

How many donuts are in 4 boxes of 12 donuts?





Jerome Bruner (1966) proposed three modes of representation:

Enactive representation (action-based)

Iconic representation (image-based)

Symbolic representation (language-based)

Effective Mathematics Teaching Practices

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Productive Beliefs

Beliefs about teaching and learning mathematics	
Unproductive beliefs	**Productive beliefs**
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Guiding Principles for School Mathematics: Curriculum

Guiding Principles for School Mathematics
Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.
Access and Equity. An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.
Curriculum. An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.
Tools and Technology. An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.
Assessment. An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.
Professionalism. In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.