

# Deficiency: Students or Environment?

In educational settings, this construction of dis/ability manifests in the double education system that splits general education and special education. Scholars have traced the ways in which special education **“serves as a vehicle for preserving general education in the midst of ever increasing diversity”** (Reid & Valle, 2004, p. 468, paraphrasing Dudley-Marling, 2001; also see Skrtic, 1991, 2005). **Rather than using research-validated frameworks** like Universal Design for Learning (UDL) and Complex Instruction (CI) **to deliver rigorous, high-cognitive demand instruction to all mathematics students**, the system of special education **shunts certain students (especially students of color) into an inferior, segregated mathematics education, thus providing a band-aid to a broken general education system and preventing larger, more systematic changes.**

Returning to the assumptions inherent in the concept of intervention, a disability studies perspective problematizes **the taken-for-granted assumption that what is “wrong” with the situation requiring intervention is a pathology or deficit within students. Instead, the problem is located in the inaccessibility of the environment; in other words, what needs to be changed is not the student, but rather the environment to allow access for students who differ from one another.** As Reid and Valle (2004) assert, “the responsibility for ‘fitting in’ has more to do with changing public attitudes and the development of welcoming classroom communities and with compensatory and differentiated instructional approaches than with individual learners (Shapiro, 1999). In other words, **our focus is on redesigning the context, not on ‘curing’ or ‘remediating’ individuals’ impairments**” (p. 468). A related line of research of working group members involves conceptualizing interventions into participation rather than content. That is, **what interventions might contribute to more equitable participation and deeper engagement across students in mathematics classrooms**? For example, one of the working group members has conducted empirical research focused on equitable participation in a Cognitively Guided Instruction algebra routine. Moreover, a political/relational model suggests that inaccessibility is embedded in the context of power relations. **Finding ways to “intervene” to make the environment accessible, then, also requires analyzing the power relations involved** in maintaining inaccessibility.

<http://jamessheldon.com/reframing-interventions-in-mathematics-education-emerging-critical-perspectives/>

# **Change the Environment**



Time: I Don't Have It



# Time: I Might Make Time to...



**Pictorial Math will save you time in the long run as it will aid students in understanding what to do and why they do it.**

- Review in the beginning of the year
- Introduce new material by re-teaching material from former grade levels
  - I am not talking about making connections to former material here.
- Reteach for full periods when they don't understand
- Reteach for full periods when they cannot remember
  - I am not talking about interleaving
- Teach multiple procedures for slightly different problems
  - I am not saying procedures are bad.



**“I’ll give you a thousand words for it.”**



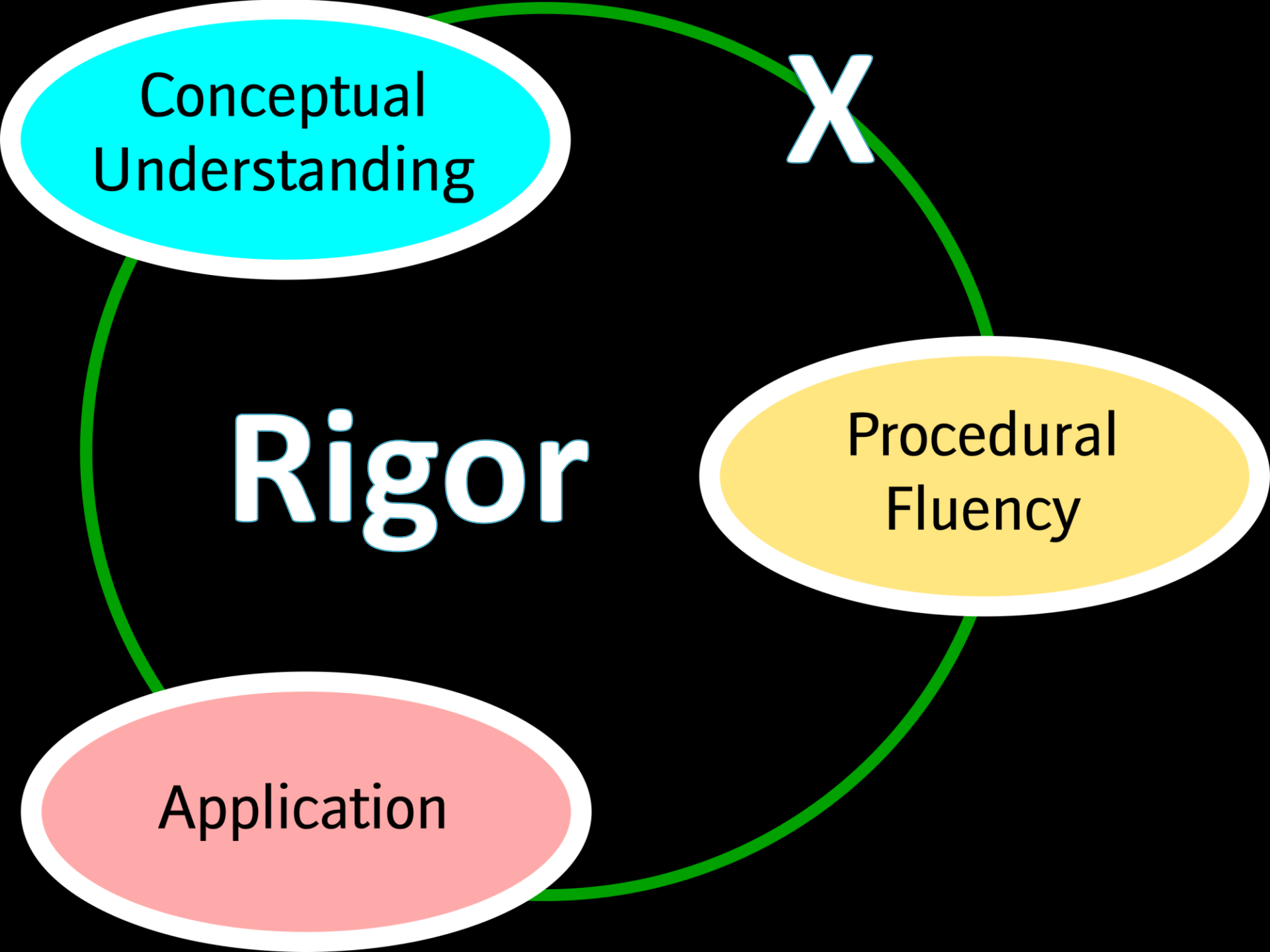
A Picture is Worth a Thousand Words  
~ Fred Barnard, 1927

The notion that a **complex idea** can be conveyed with just a **single still image** or that **an image of a subject conveys its meaning or essence more effectively** than a description does.

# A Picture is Worth a Thousand Words

~ Fred Barnard, 1927

- **"Hearing something a hundred times isn't better than seeing it once"**  
~ Chinese Expression
- **"The drawing shows me at a glance what would be spread over ten pages in a book."** ~ Ivan Turgenev, Russian Author
- **"A good sketch is better than a long speech"** ~ Napoleon Bonaparte, Emperor of the French





# Base 10 Blocks for WHOLE NUMBERS

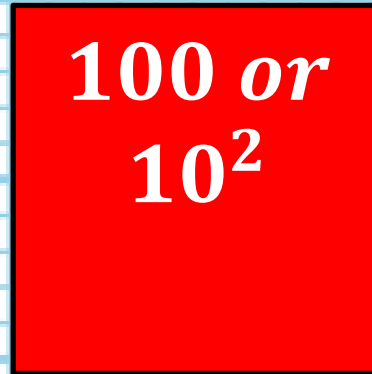
1



10



100 *or*  
 $10^2$



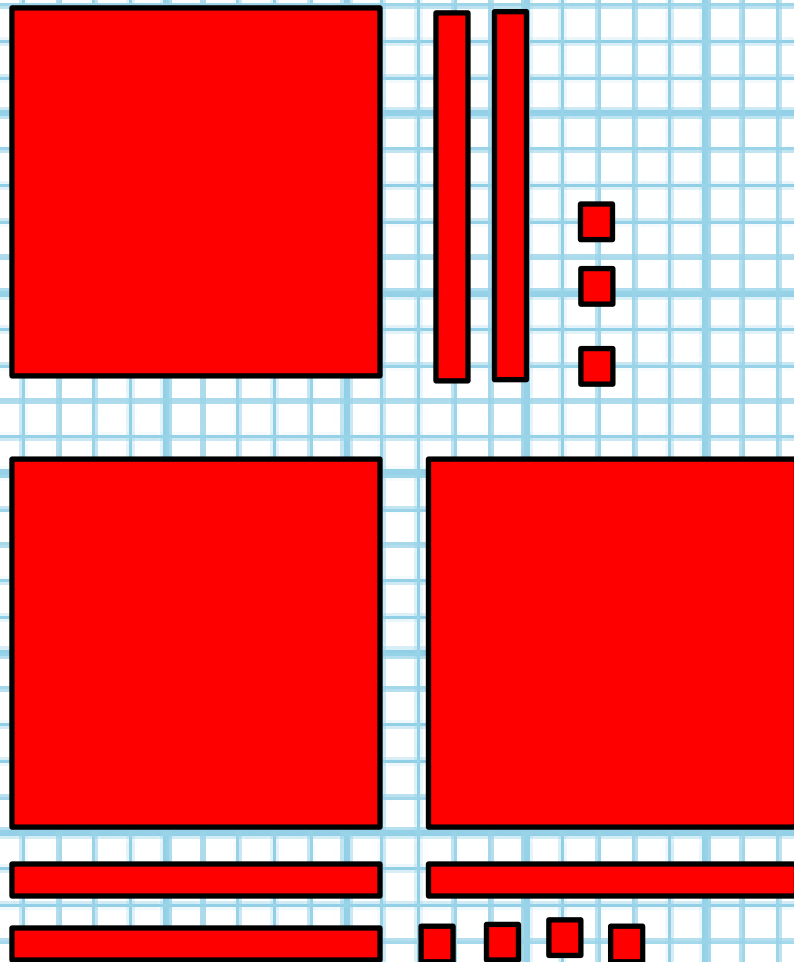
You have to define what one is.

In the case of whole numbers, one is the small square, which makes the others ten and one hundred.

Begin referring to 100 and “ten squared” or ten in two dimensions, length and width.

# Base 10 Block ADDITION (Whole Numbers)

Use a blank sheet of paper as your work space. The sum is on the paper, and the unused tiles are off the paper.



$$[ 123 ] + [ 234 ]$$

$$[ 1(10^2) + 2(10) + 3(1) ]$$

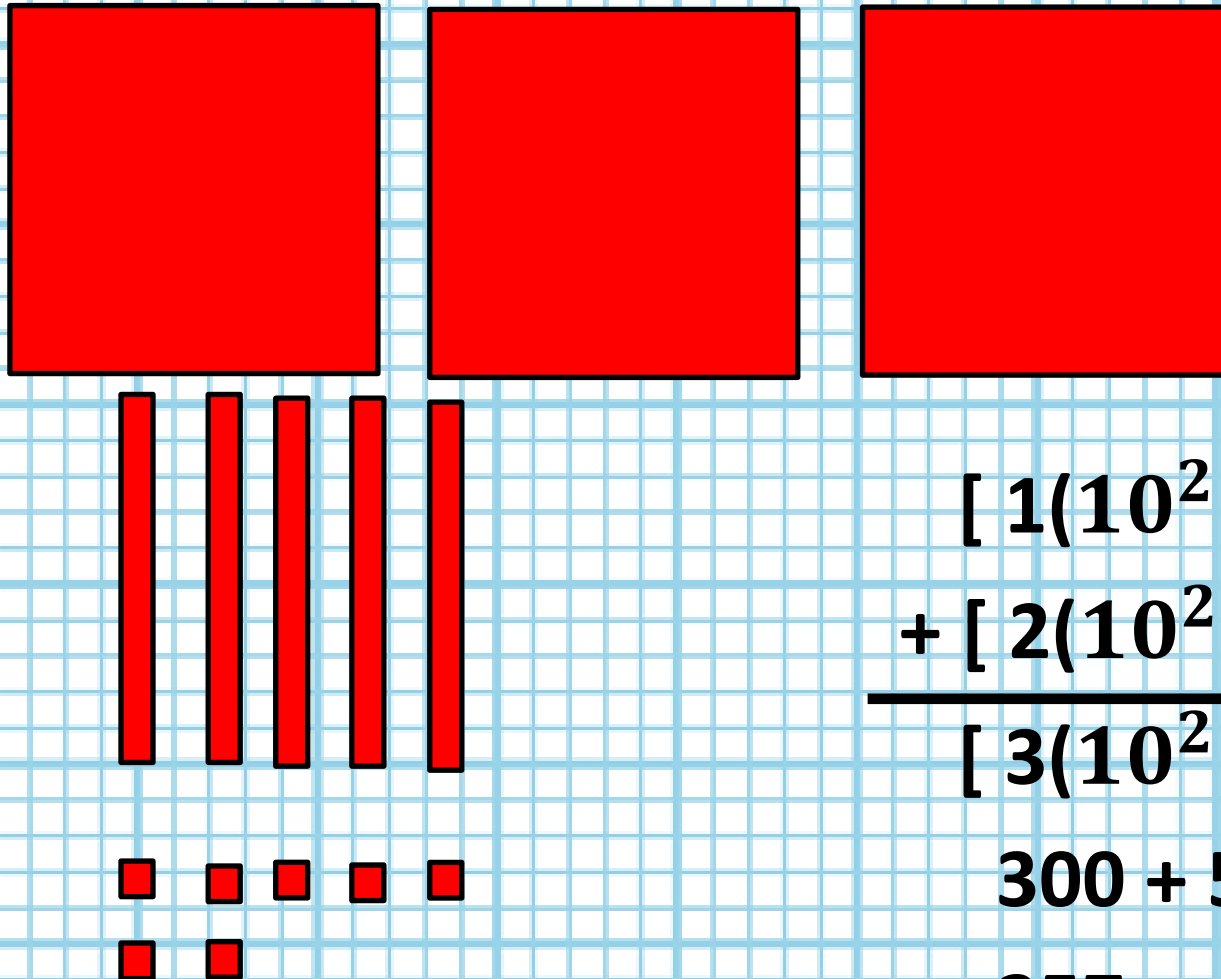
$$+ [ 2(10^2) + 3(10) + 4(1) ]$$

---

# Base 10 Block ADDITION (Whole Numbers)

Have them  
put the  
same  
shapes  
together.

$$[ 123 ] + [ 234 ]$$



$$[ 1(10^2) + 2(10) + 3(1) ]$$

$$+ [ 2(10^2) + 3(10) + 4(1) ]$$

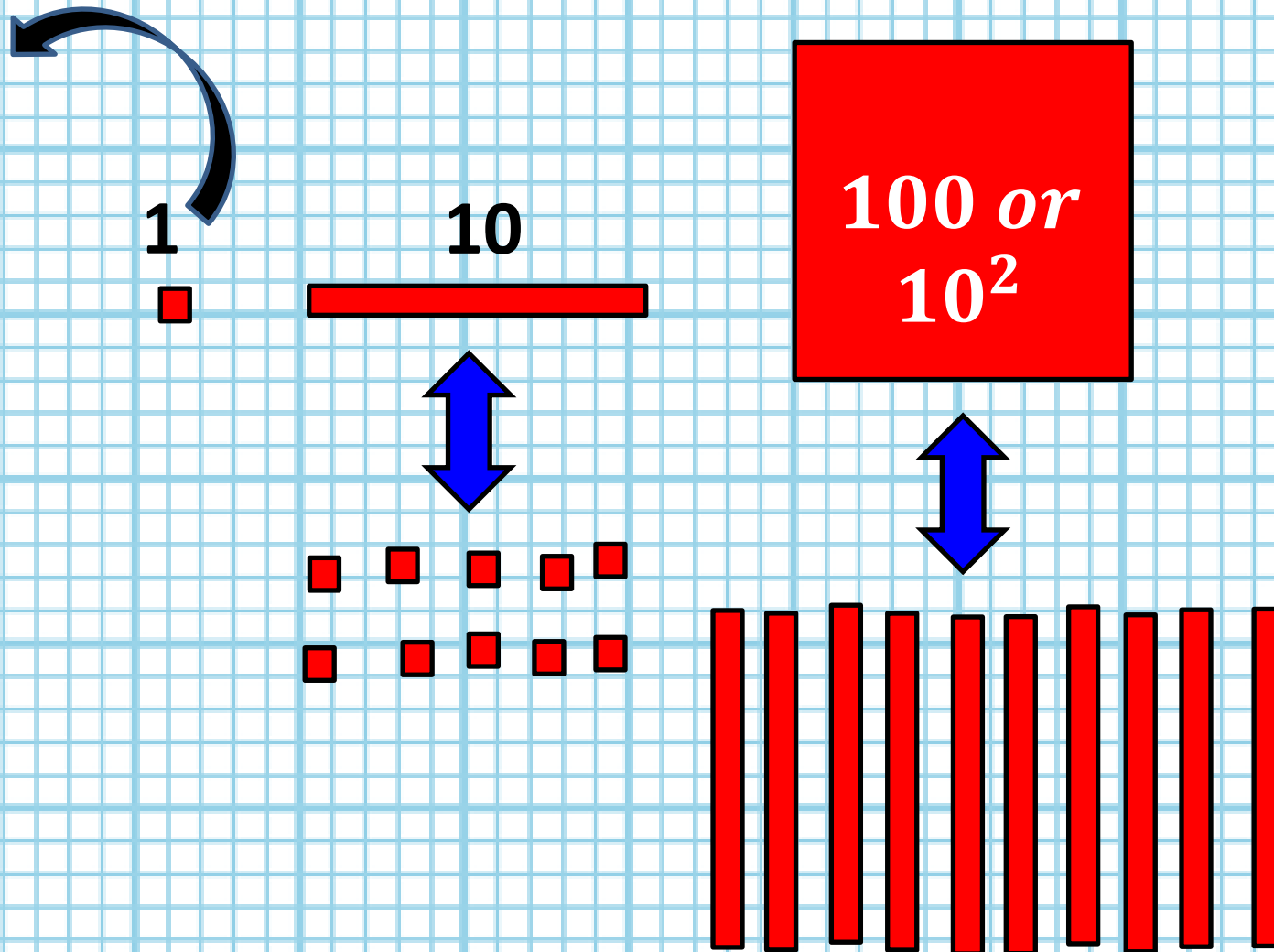
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$$[ 3(10^2) + 5(10) + 7(1) ]$$

$$300 + 50 + 7$$

$$357$$

# Base 10 Block SUBTRACTION & REGROUPING (Whole Numbers)



You simply remove tiles from the work space for subtraction.

Addition and subtraction in base ten may require regrouping, changing a ten to 10 ones or a hundred to 10 tens or vice versa.

This should be done before connecting (teaching) to the standard algorithm.

# Base 10 Blocks for DECIMALS

**.01**



**.10**



**1.00 or  
1.0<sup>2</sup>**



You have to define what one is.

In the case of decimals, one is the large square, which makes the others one tenth and one hundredth.

Begin referring to 100 and “ten squared” or ten in two dimensions, length and width.

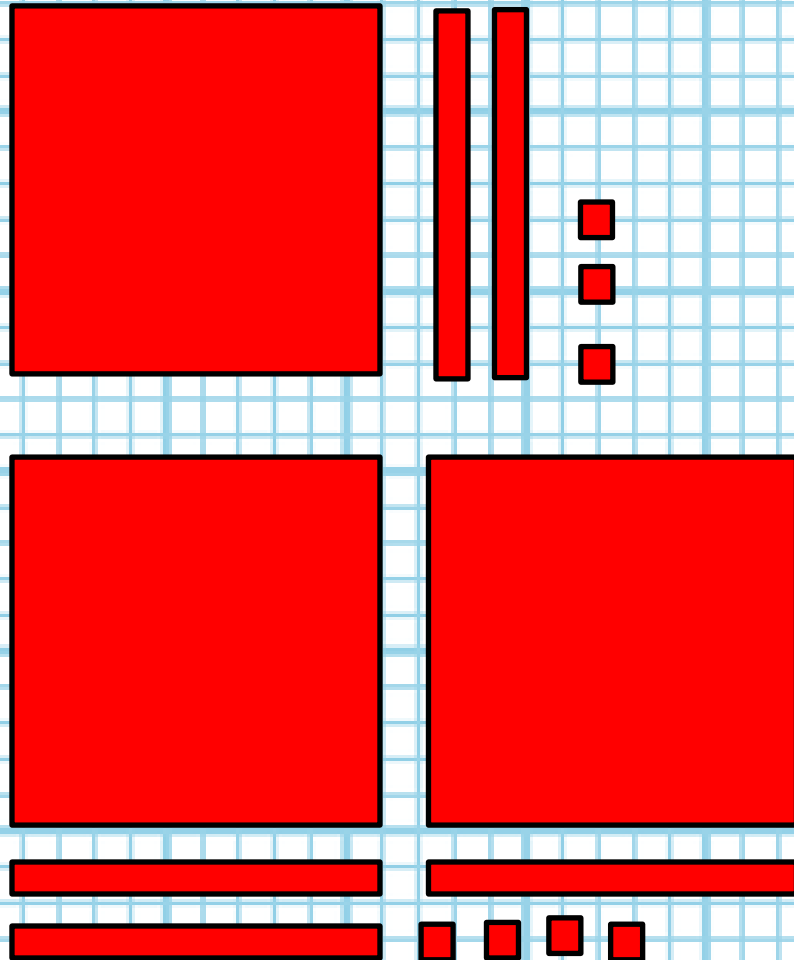
***Notice, I have written 1 as 1.00 this time to help students see there are one hundred hundredths in one.***

***Similarly, there are ten one hundredths in one tenth.***

***Also, (1.0)(1.0) = 1.00***

# Base 10 Block ADDITION (Decimals)

Use a blank sheet of paper as your work space. The sum is on the paper, and the unused tiles are off the paper.



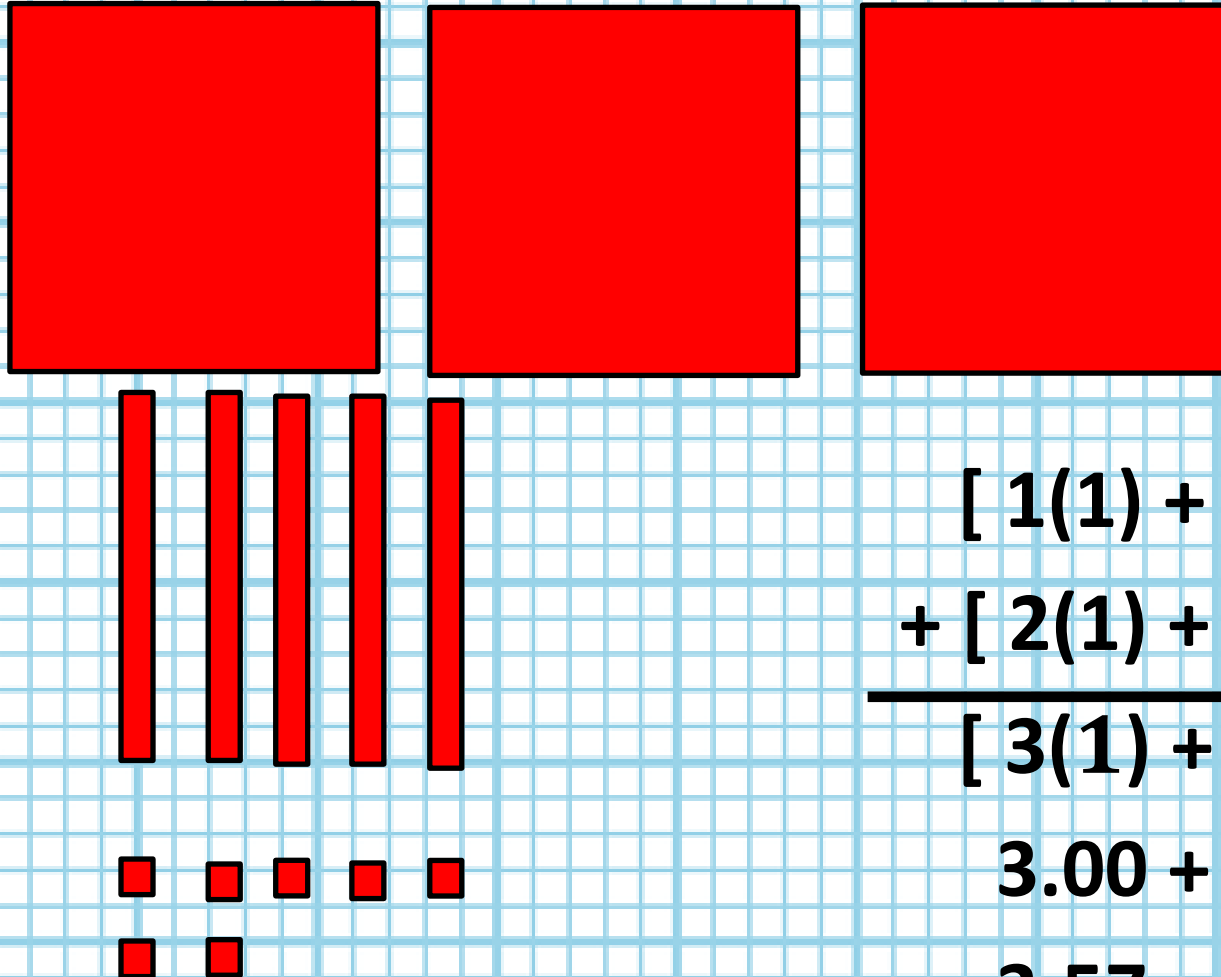
$$\begin{array}{r} [ 1.23 ] + [ 2.34 ] \\ [ 1(1) + 2(. 1) + 3(. 01) ] \\ + [ 2(1) + 3(. 1) + 4(. 01) ] \\ \hline \end{array}$$



# Base 10 Block ADDITION (Decimals)

Have them  
put the  
same  
shapes  
together.

$$[ 1.23 ] + [ 2.34 ]$$



$$[ 1(1) + 2(.1) + 3(.01) ]$$

$$+ [ 2(1) + 3(.1) + 4(.01) ]$$

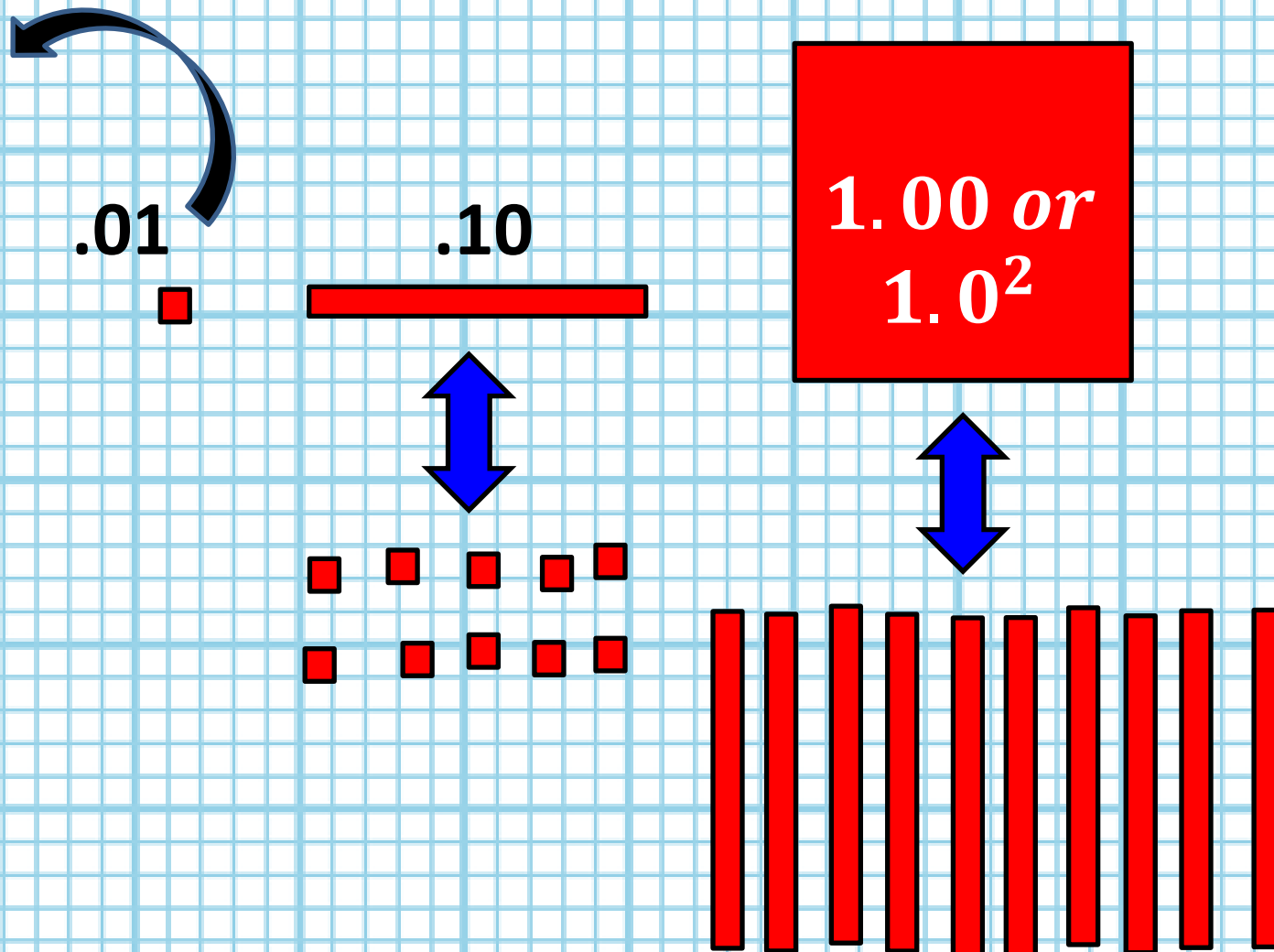
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$$[ 3(1) + 5(.1) + 7(.01) ]$$

$$3.00 + .50 + .07$$

$$3.57$$

# Base 10 Block SUBTRACTION & REGROUPING (Decimals)



You simply remove tiles from the work space for subtraction.

Addition and subtraction in base ten may require regrouping, changing 10 tenths to a one or 10 hundredths to a ten or vice versa.

The shapes demonstrate why you line up the decimals during the standard algorithm and should be done 1<sup>st</sup>

# Difference Between Whole Number and Decimals?

- While the notation may be different, we did essentially the same thing.
- TURN THEM UPSIDE DOWN AND NOW YOU HAVE ALGEBRA TILES !
- Algebra is the generalization of any base or any side length

# Base 10 Blocks as ALGEBRA TILES

**1**



***Constant***

**$1x$**



***Linear –  
a line of  
length  $x$   
and a  
height of 1***

**$1x^2$**



***Quadratic –  
a square  
with a side  
length of  $x$ .***

In the case of algebra tiles, one is the small square, which is the same as base ten. You might say it is constantly one. Really it is a constant, because the value of  $x$  will not influence it.

Begin referring to  $x$  squared as a square with a side length of  $x$ . A square with two dimensions, a length of  $x$  and a width of  $x$ .

# ALGEBRA TILES

$$1x^0$$



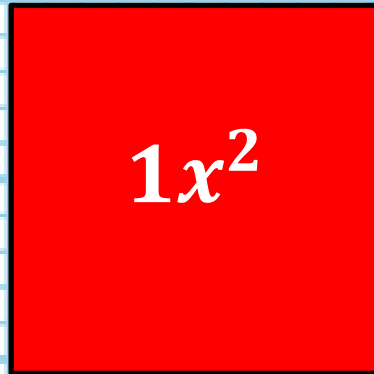
***Constant***

$$1x^1$$



***Linear –  
a line of  
length  $x$   
and a  
height of 1***

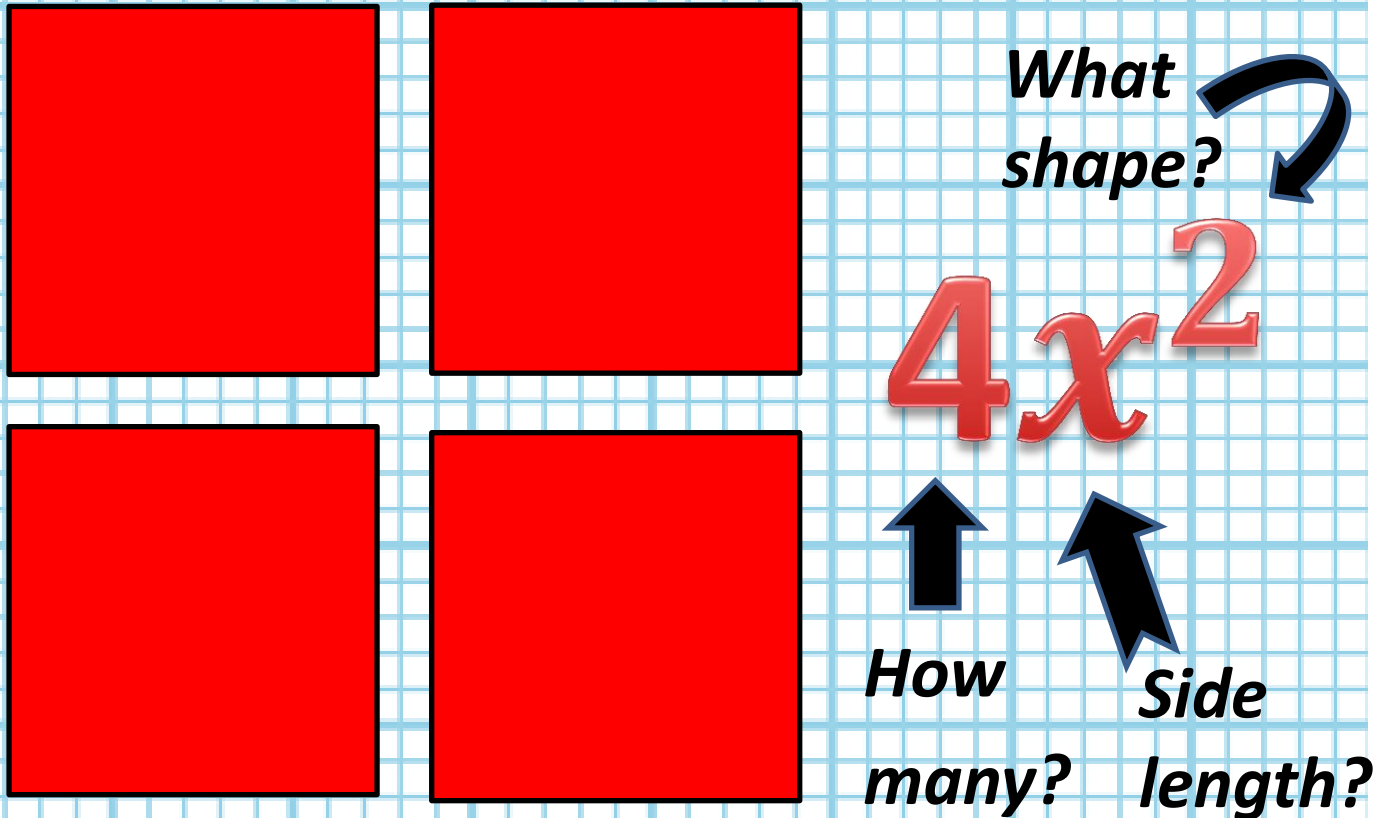
$$1x^2$$



***Quadratic–  
a square  
with a side  
length of  $x$ .***

Our number system is based on powers of ten; therefore, it is the exponent, which drives what shape it is.

# Anatomy of an Algebraic Term



You want students to discover this and/or be able to tell you when you ask:

What does the exponent mean?

What does the base mean?

What does the coefficient mean?

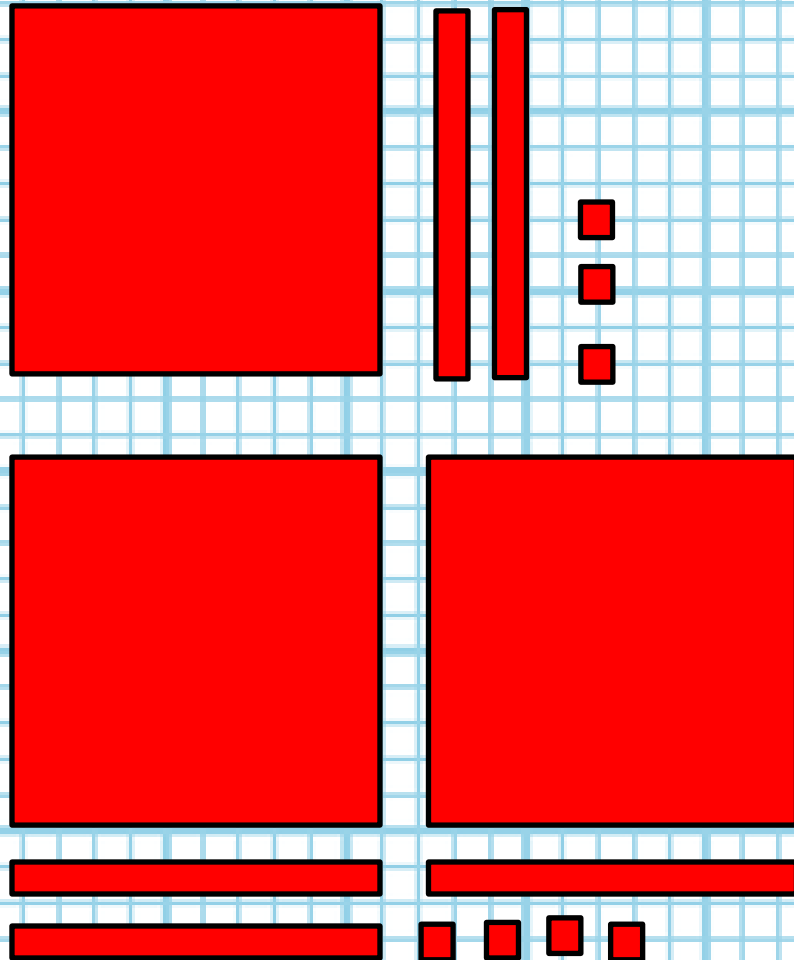


# Algebra Tile ADDITION

## (Any Base)

$$(1x^2 + 2x + 3) + (2x^2 + 3x + 4)$$

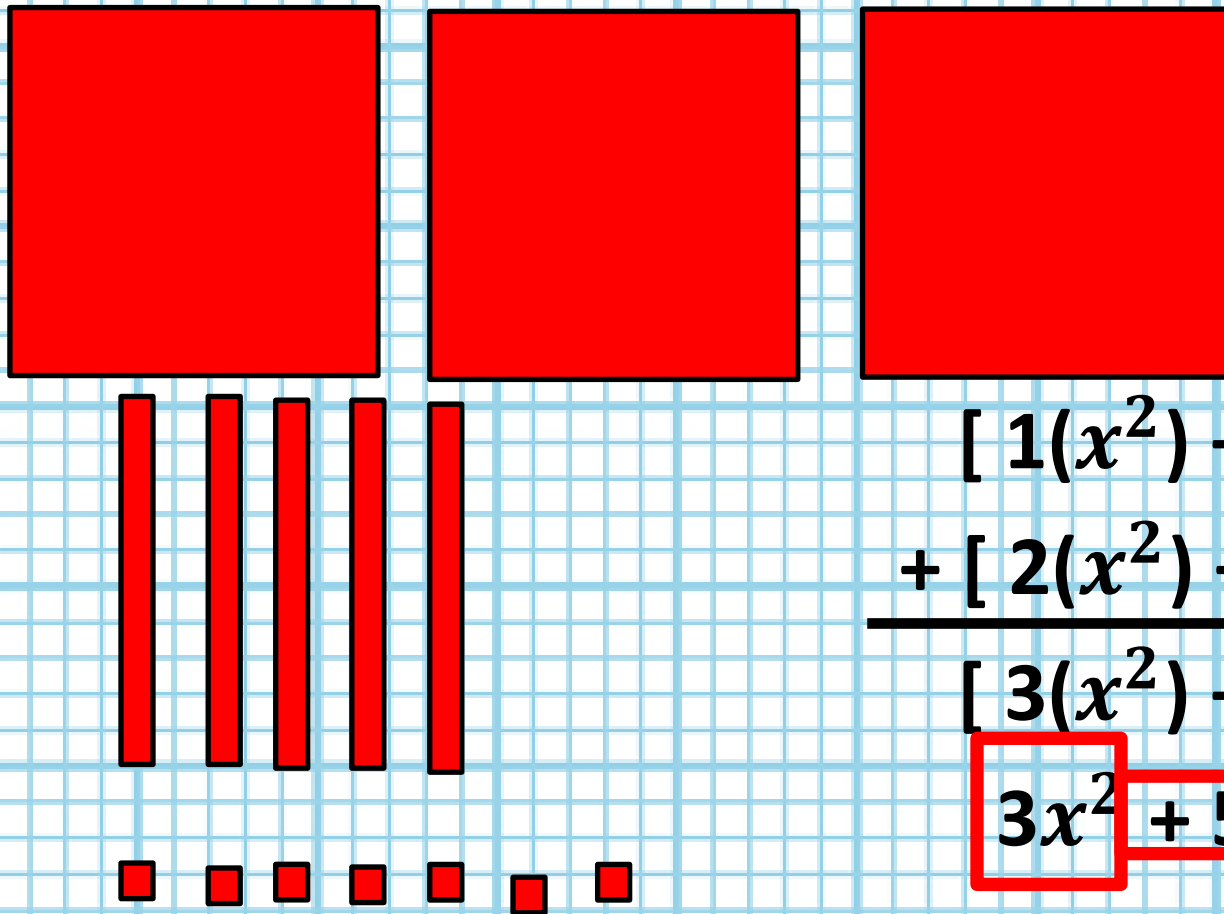
Use a blank sheet of paper as your work space. The sum is on the paper, and the unused tiles are off the paper.



$$\begin{array}{r} [ 1(x^2) + 2(x) + 3(1) ] \\ + [ 2(x^2) + 3(x) + 4(1) ] \\ \hline \end{array}$$

# Algebra Tile ADDITION

$$(1x^2 + 2x + 3) + (2x^2 + 3x + 4)$$



Have them  
put the same  
shapes  
together, aka  
combine like  
terms.

$$[ 1(x^2) + 2(x) + 3(1) ]$$

$$+ [ 2(x^2) + 3(x) + 4(1) ]$$

---

$$[ 3(x^2) + 5(x) + 7(1) ]$$

$$3x^2 + 5x + 7$$

*Can you put the terms together? No, they are not the same shape.  
Squares, Lines & Dots or Quadratic Terms, Linear Terms & Constants*

# Algebra Tile REGROUPING?

## (What Base?)

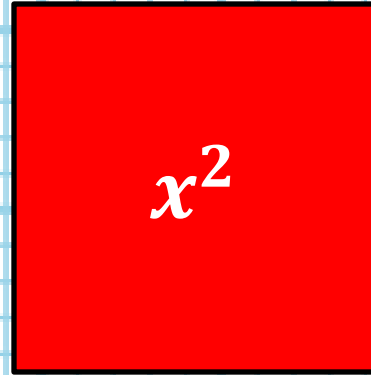
1



$x$



$x^2$



*“My students aren’t ready for algebra.”*

*“They have to master basic skills first before learning algebra.”*

***While it is preferred to see algebra as a generalization of any base, could we use algebra to remediate number? Instead of going specific to general could we go general to specific?***

***Keep students on grade level and remediate number with Tier Two interventions using more time to relate number to algebra. In class, use polynomial addition & subtraction to remediate integers or even whole numbers. Remediate in context of grade level material!***

There is no regrouping with algebra tiles as you do not know what the base is, or how many units it takes to make the units of the next power.

Consider this. The lack of regrouping actually makes the cognitive demand for learning algebra less than the cognitive demand for number when using algebra tiles and base ten blocks.

# Next Level of Abstraction

## *Drawing the Tiles Under the Expressions*

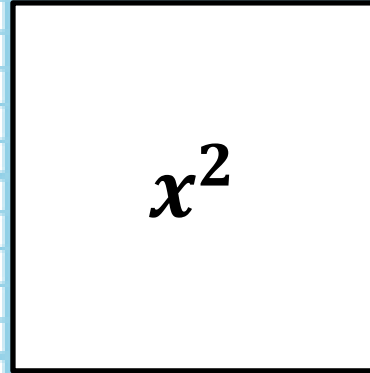
1



$x$



$x^2$



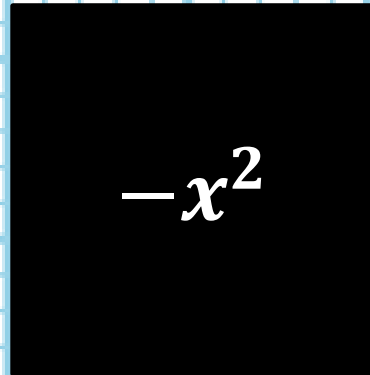
-1



$-x$



$-x^2$



$w$

“Open” is positive.

“Closed” is negative.

Referring to them as white being positive and black being negative is not a good idea.

# Integer SUBTRACTION

$$(4) - (3) =$$

*T: What is 4-3?*

*Ss: 1*

$$(4) - (3) = 1$$

*T: So, why does this look like 7?*

*Ss: You have to change them to negatives.*

*T: Like this?*

*Ss: No, change all of them!*

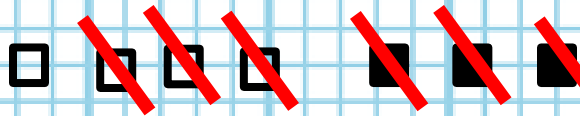
*T: Like this?*

*Ss: No, no. Change ALL of them!*

*T: Oh, you mean I have to change ALL of them? All of minuend or subtrahend?*

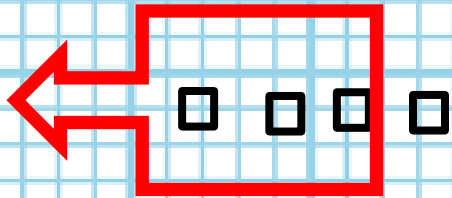
*Ss: All of the subtrahend.*

After integers, my students are familiar with zero pairs. Before subtracting polynomials, I give them this problem.

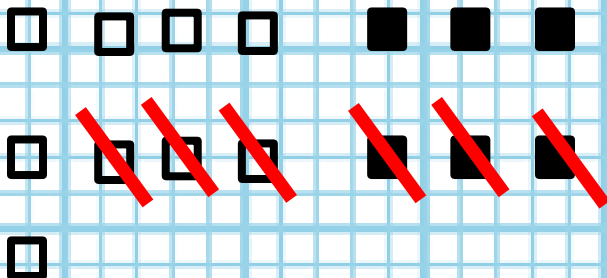


# Integer SUBTRACTION

$$4 - 3 = 1$$



$$+4 - 3 = 1$$



By the way, you want them to go from seeing this (above) to seeing this (below).

So they can simplify without the interference of the subtraction concept.

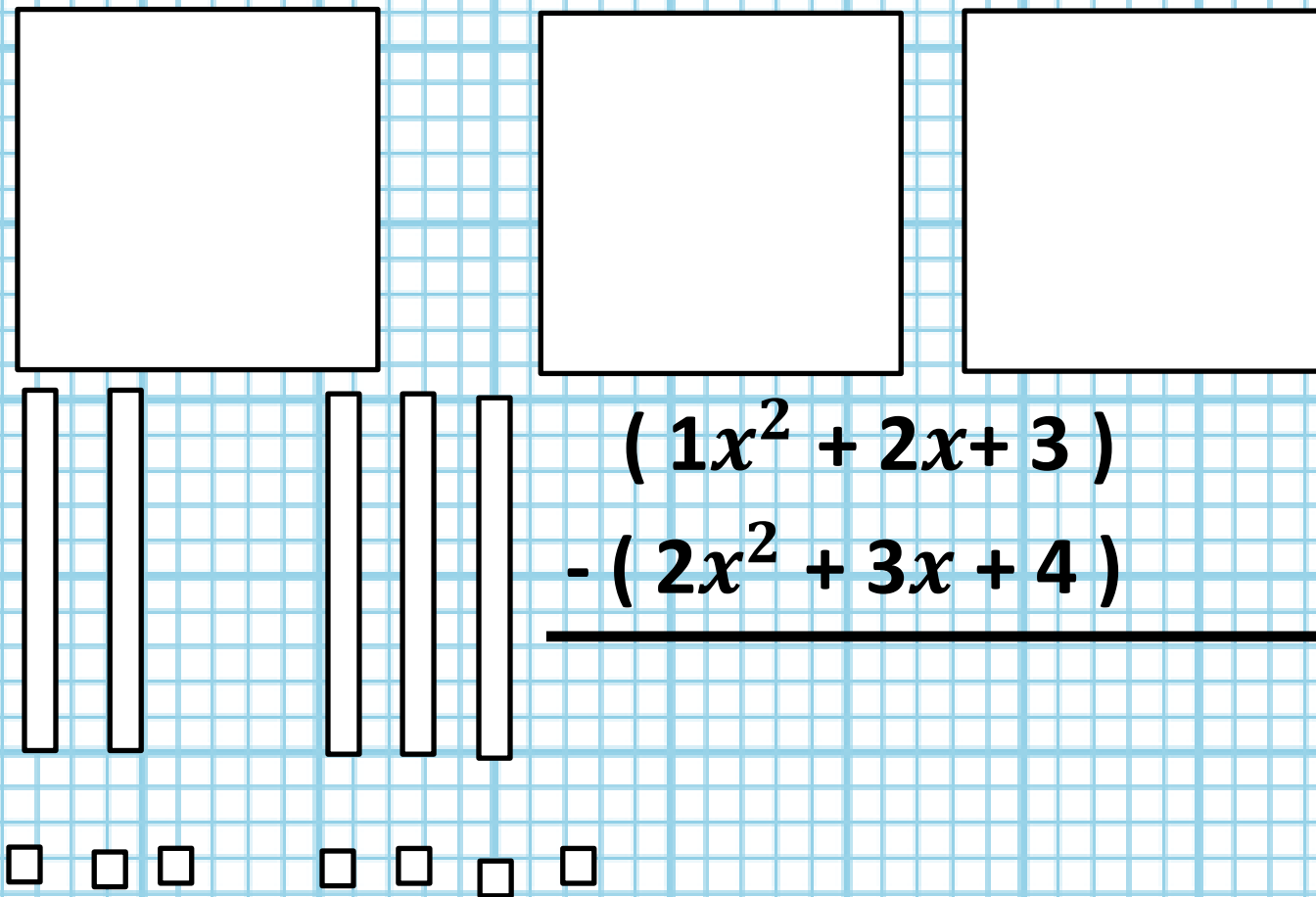
You'll see later why I ask my students if it is OK to use dots instead of squares when drawing them out.

If you think about it, then they really aren't supposed to have any dimension to them as they are  $x^0$  anyway.



# Algebra Tile SUBTRACTION 1

$$(1x^2 + 2x + 3) - (2x^2 + 3x + 4)$$



***THIS IS NOT  
CORRECT!***

Negative means opposite so which do we switch, the minuend or the subtrahend?

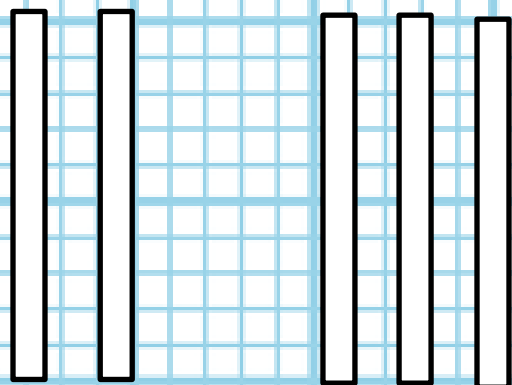
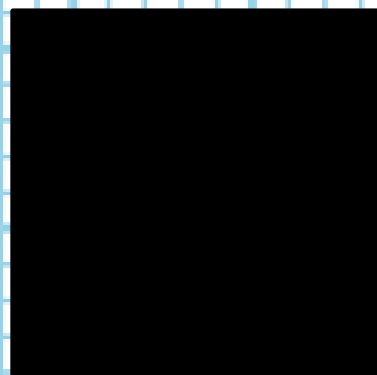
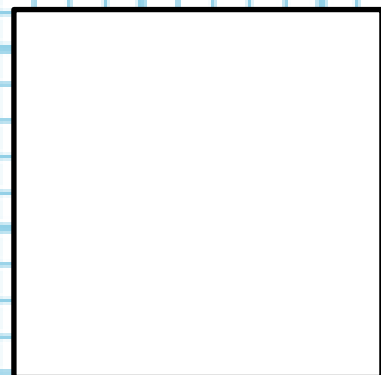
Which did we switch with 4-3?

Do we change one of them or all of them?

At some point, you can point out this is the equivalent to distributing a negative one.

# Algebra Tile SUBTRACTION 1

$$(1x^2 + 2x + 3) - (2x^2 + 3x + 4)$$



$$(1x^2 + 2x + 3)$$

$$- (2x^2 + 3x + 4)$$

---

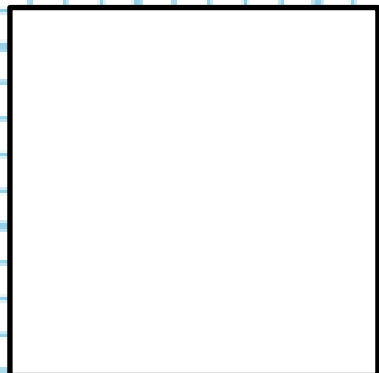
***THIS IS NOT  
CORRECT YET***

What is the  
opposite of  
 $2x^2$ ?

$-2x^2$  so shade  
it in.

# Algebra Tile SUBTRACTION 1

$$(1x^2 + 2x + 3) - (2x^2 + 3x + 4)$$



$$(1x^2 + 2x + 3)$$

$$- (2x^2 + 3x + 4)$$

---

***THIS IS NOT  
CORRECT YET***

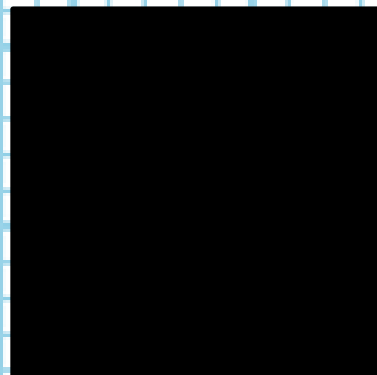
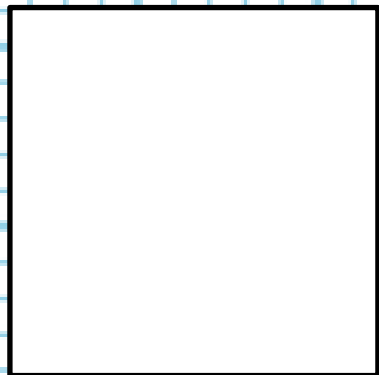
What is the  
opposite of

**$+ 3x$** ?

**$- 3x$**  so shade  
it in.

# Algebra Tile SUBTRACTION 1

$$(1x^2 + 2x + 3) - (2x^2 + 3x + 4)$$



$$(1x^2 + 2x + 3)$$

$$- (2x^2 + 3x + 4)$$

---

***THIS IS NOT  
CORRECT YET***

What is the  
opposite of  
 $+ 4$ ?

$- 4$  so shade it  
in.

# Algebra Tile SUBTRACTION 2

$$(1x^2 + 2x + 3) + (-2x^2 - 3x - 4)$$

Diagram illustrating the subtraction of  $(-2x^2 - 3x - 4)$  from  $(1x^2 + 2x + 3)$  using algebra tiles. The tiles are arranged in two rows, with red diagonal lines indicating the removal of the second polynomial's tiles from the first polynomial's tiles.

Row 1:  $(1x^2 + 2x + 3)$   
 $+ (-2x^2 - 3x - 4)$

---

Row 2:  $-1x^2 + (-1x) + (-1)$




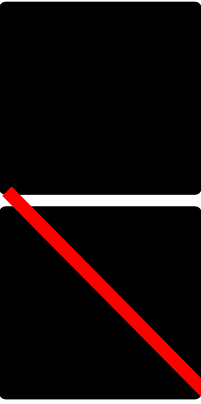
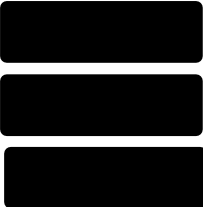


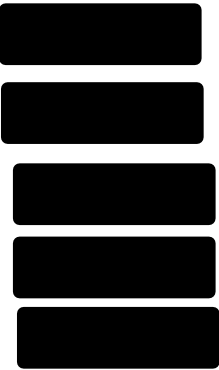
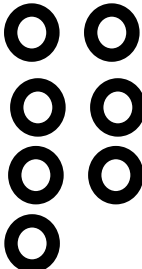
Row 3:  $-1x^2 - 1x - 1$

From our work with integers, students know that adding the opposite is the same as subtracting by doing problems like four minus three then doing four plus a negative three.

*If students make the simplification error in the sum/difference, then use it to review integers, "What is the opposite of a positive?" It is really an error showing their understanding. "Can you simplify that?"*

# Combining Expressions

## Three Column Notes

	+		=			
$1x^2 - 2x + 3$		$-2x^2 - 3x + 4$		$-1x^2 - 5x + 7$		
						
						
						

*Require students to draw the shapes AND write the symbols.*



# Combining Expressions

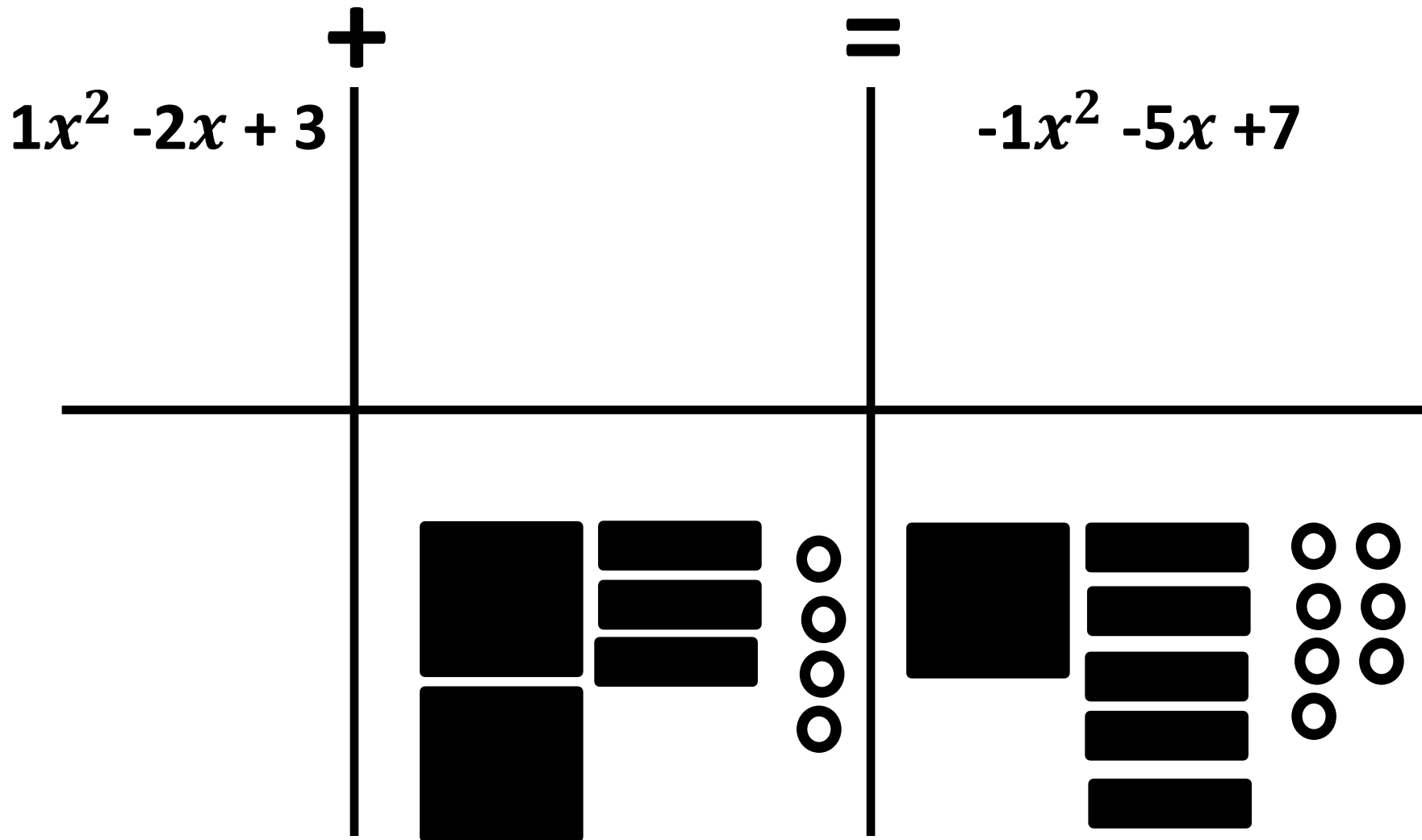
## Three Column Notes

	+	=
$1x^2 - 2x + 3$		$-2x^2 - 3x + 4$
<div><div></div><div><div></div><div></div></div><div><div></div><div></div><div></div></div></div>	<div><div></div><div></div></div> <div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div></div>

*Eliminate just enough information to make it doable.  
Require students to draw the shapes AND write the symbols.*

# Combining Expressions

## Three Column Notes



*Eliminate just enough information to make it doable.  
Require students to draw the shapes AND write the symbols.*

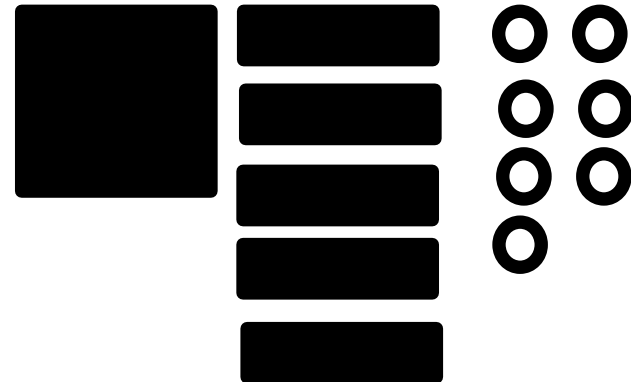
# Combining Expressions

## Three Column Notes

+

=

$-1x^2 - 5x + 7$



*Make your own problem.*

*Require students to draw the shapes AND write the symbols.*

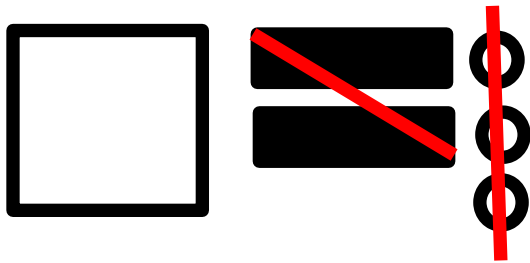
# Combining Expressions

## Three Column Notes

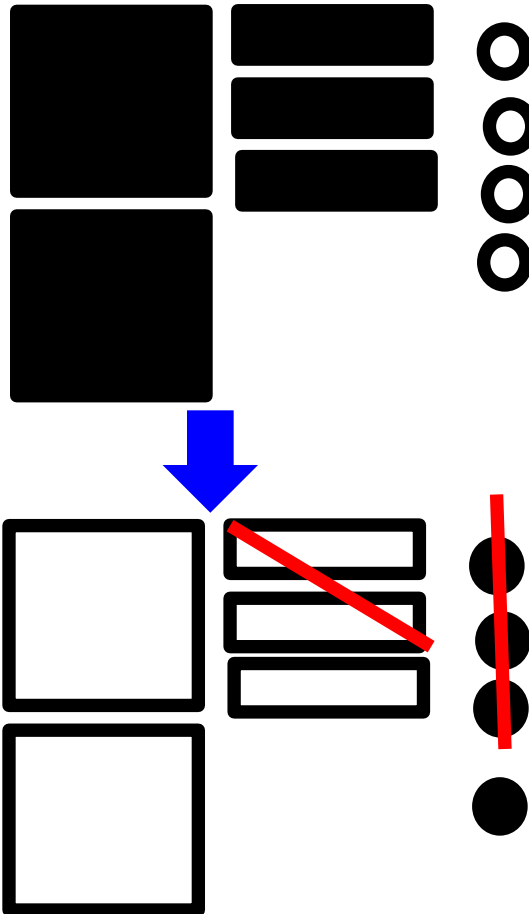
-

=

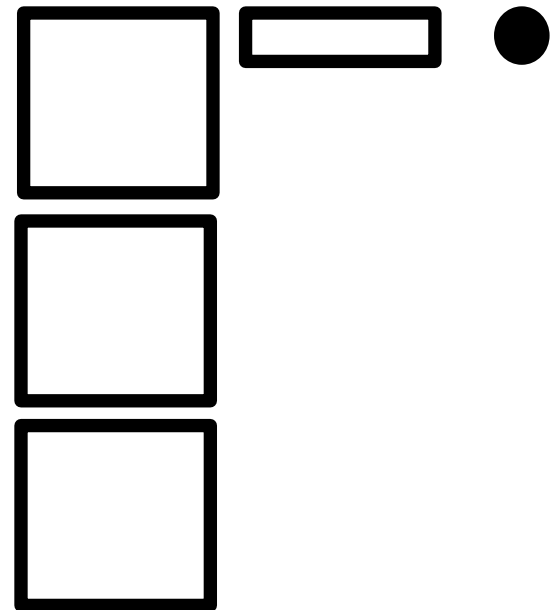
$$1x^2 - 2x + 3$$



$$-2x^2 - 3x + 4$$



$$3x^2 + 1x - 1$$

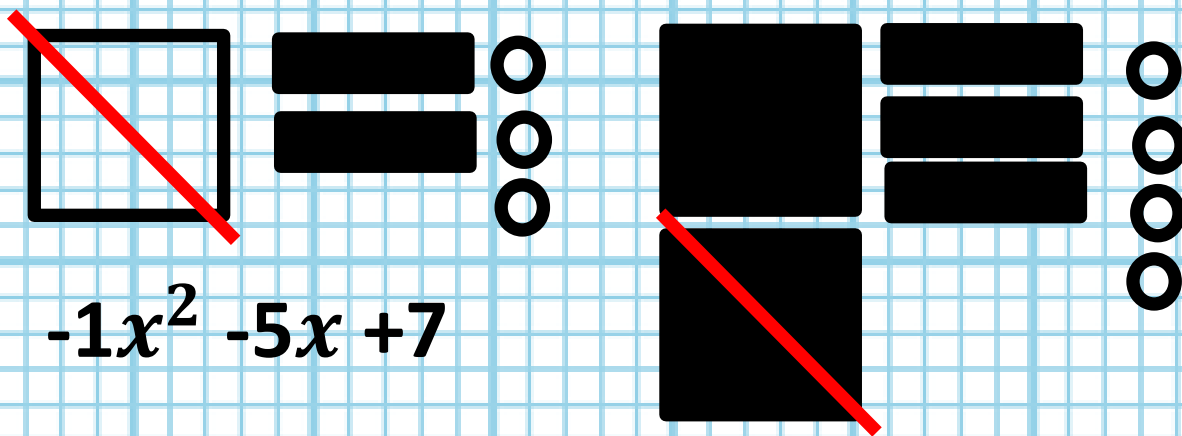


*If you do subtraction, then make them change the shapes to the opposite before combining.*

# Next Level of Abstraction

## Drawing Around the Symbols

$$(1x^2 - 2x + 3) + (-2x^2 - 3x + 4)$$



$$-1x^2 - 5x + 7$$

$$(11x^2 + 32x + 13) + (22x^2 - 34x + 14)$$

$$33x^2 - 2x + 27$$

*This is a good time to introduce larger coefficients or even fractional coefficients if you need to remediate fractions or decimals. Things get squishy.*

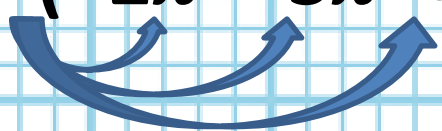
THE HORIZONTAL METHOD IS GOOD FOR DRAWING SHAPES AROUND OR BELOW.

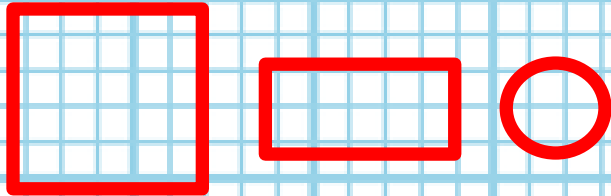
Make your squares, quadratic, large and your rectangles, linear, smaller, but with the same side length.

Constants get circles instead of the little squares that are more difficult to draw and easily confused with quadratics.

# Final Level of Abstraction

## Line Up Like Terms Vertically

$$\begin{array}{r} (1x^2 - 2x + 3) - (-2x^2 - 3x + 4) \\ +2x^2 + 3x - 4 \\ \hline 3x^2 + 1x - 1 \end{array}$$




***Keep asking, "Can I simplify?"***  
***No, quadratic, linear & constant***

THE VERTICAL METHOD IS GOOD FOR KEEPING LIKE TERMS TOGETHER AND REWRITING SUBTRACTION.

You can reference the shapes as you do it. You want students to follow the procedure of combining like terms based on what they learned about the areas they represent.

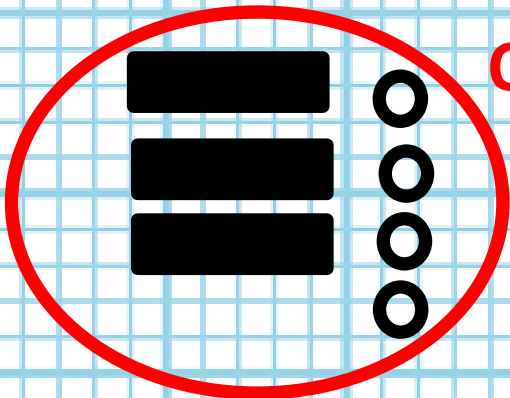
You can redraw them at the end to reinforce it.

# Distributive Property

*Multiple Groups of Expressions*

# Distributive Property

2  $(-3x + 4)$  Read, "Two groups of negative three x plus four or two groups of negative three times a number plus four." Drawn as:



One group

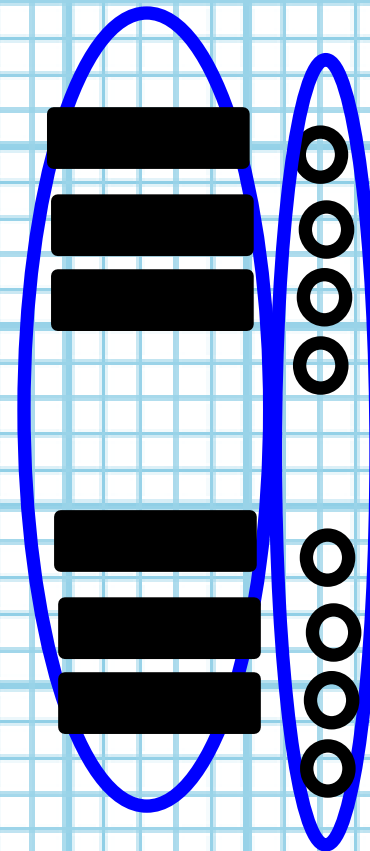
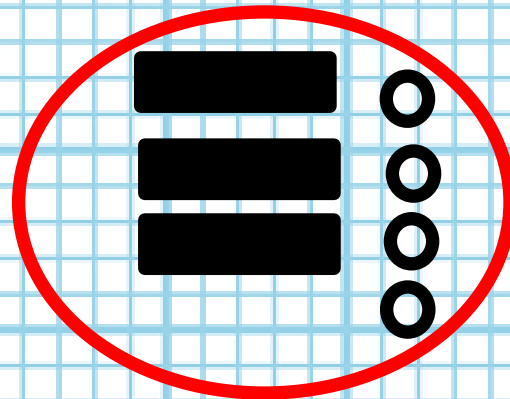
Two groups

How many  
negative x's?

How many  
constants?

So, the  
product is?

$-6x + 4$



Most students haven't had the conceptual instruction for multiplication.

Most can multiply, but some do not realize it is making groups.

Others can tell you it is making groups, but it still does not inform their mathematics. For example, they don't make groups to simplify.

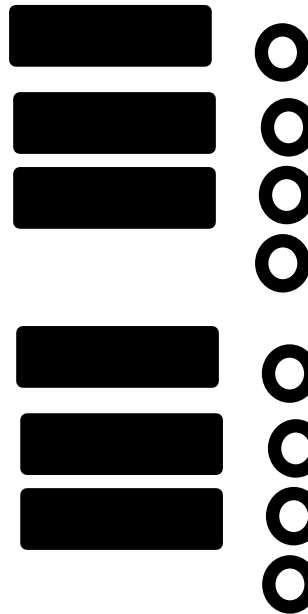
This needs to be reinforced when distributing.



# Distributive Property

## Three Column Notes

$$2(-3x + 4)$$

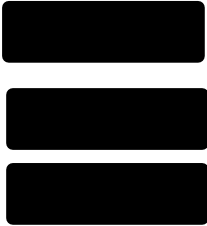

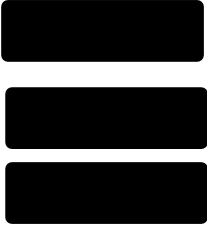



two groups of  
negative three times a  
number plus four

Three groups of two  
times a number minus  
five

# Distributive Property

## Three Column Notes

		
		
$2(-3x + 4)$		

# Substitution

*Peas in a Pod*

# Context for Substitution

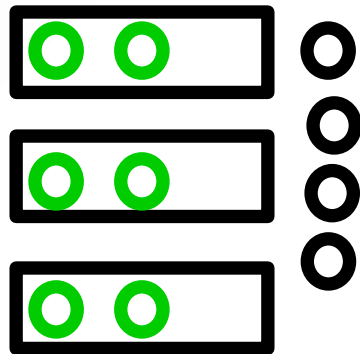


- How are the peas arranged in a pod?
- You can think of a pea pod as a linear term.
- Let  represent the peapod.
- Let  represent the peas

If  $p = \#$  of peas in a pod,  
then..



# Context for Substitution



If  $p$  = # of peas in a pod,  
then..

How many total peas if  
the pea pods hold **two**  
peas each?

**How did you do it?**

$$3p + 4$$

$$3(\mathbf{2}) + 4$$

$$\mathbf{6} + 4$$

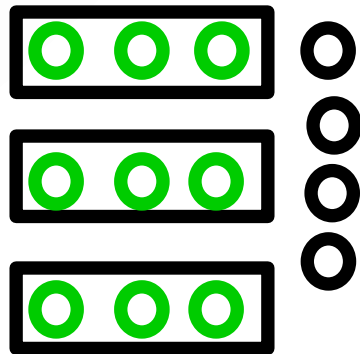
$$10$$



# Context for Substitution

If  $p = \#$  of peas in a pod,  
then..

How many total peas if  
the pea pods hold **three**  
peas each?



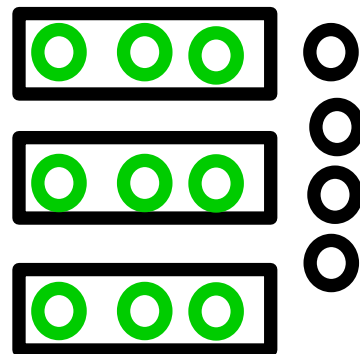
$$2(3p + 4)$$

$$6p + 8$$

$$6(\mathbf{3}) + 8$$

$$\mathbf{18} + 8$$

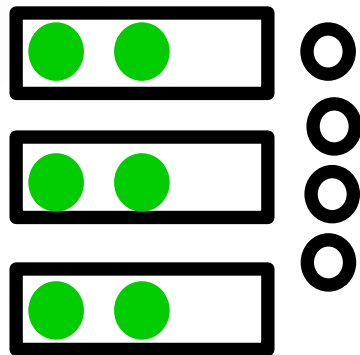
$$26$$





# Context for Substitution

## Negative Values (not great)



Wouldn't it be weird to open a peapod and find a note saying you owe peas?

If  $p$  = # of peas in a pod, then..

Would you owe or have peas if the pea pods hold **negative two** peas each? How many?

$$3p + 4$$

$$3(-2) + 4$$

$$-6 + 4$$

-2, You would owe two peas.



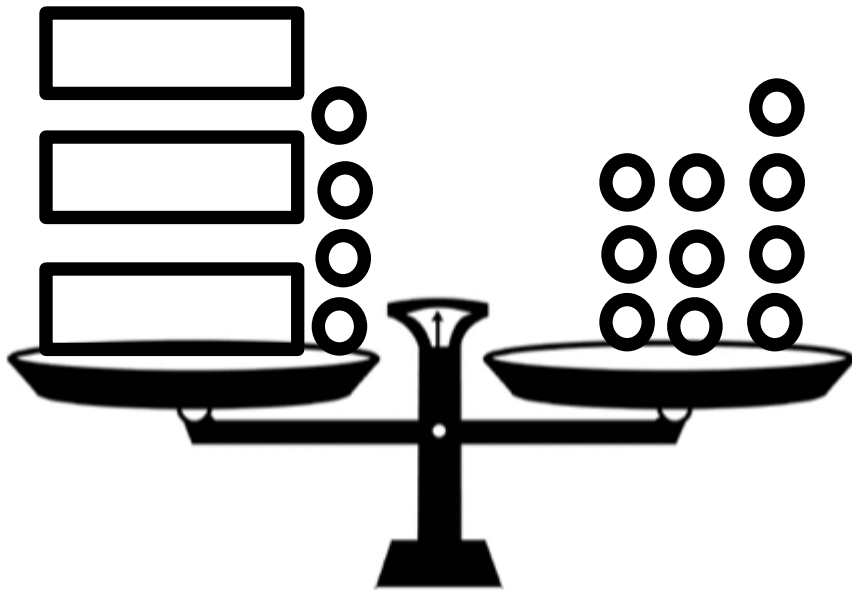
# Context for Equations

If  $p$  = # of peas in a pod,  
then..

How many peas in the  
each pea pod? (Disregard  
the weight of the pod.)

How did you do it?

$$3p + 4 = 10$$







# Context for Equations

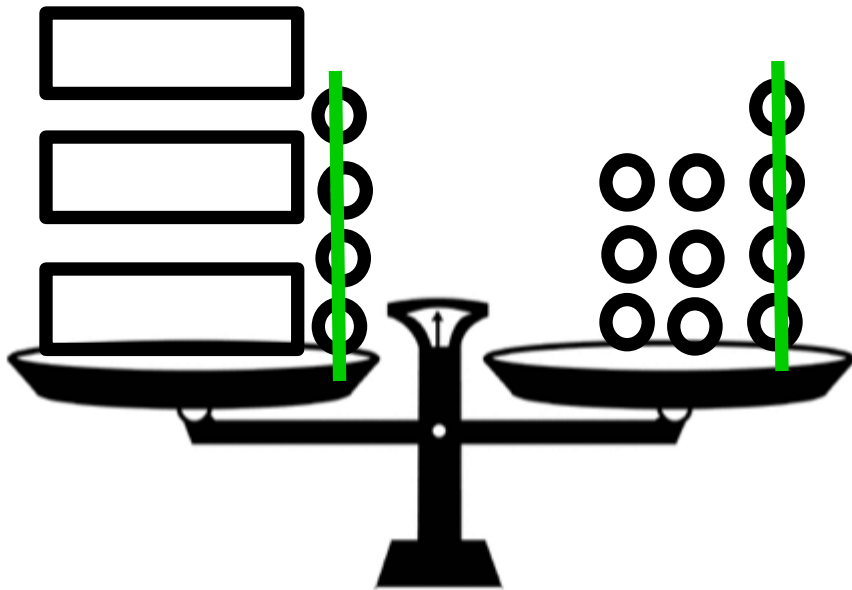
If  $p$  = # of peas in a pod,  
then..

How many peas in the  
each pea pod? (Disregard  
the weight of the pod.)

How did you do it?

$$3p + 4 = 10$$

$$3p + 4 - 4 = 10 - 4$$





# Context for Equations

If  $p$  = # of peas in a pod,  
then..

How many peas in the  
each pea pod? (Disregard  
the weight of the pod.)

How did you do it?

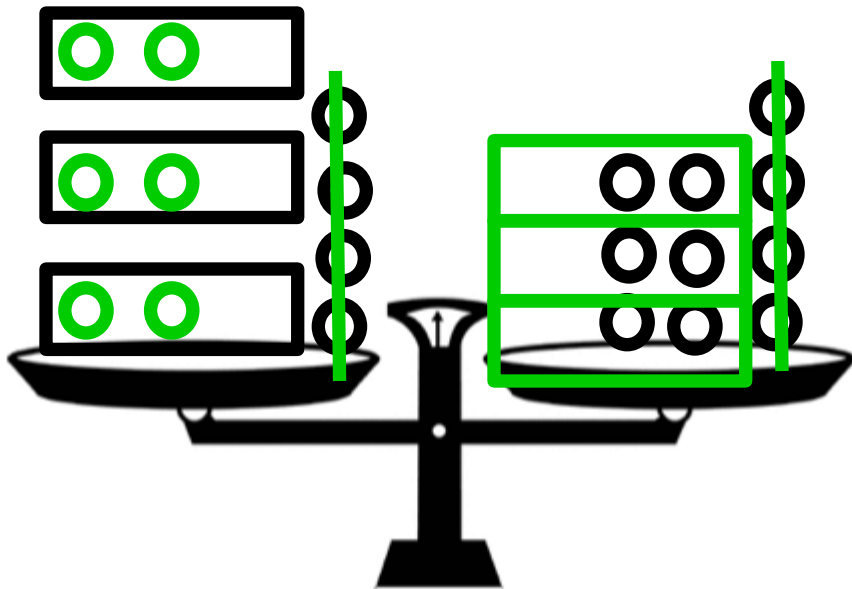
$$3p + 4 = 10$$

$$3p + 4 - 4 = 10 - 4$$

$$3p = 6$$

$$\frac{3}{3} p = \frac{6}{3}$$

$$p = 2$$



# Equations

*Two Groups of Equivalent  
Expressions*

# Equations: Misconceptions

- I have to “scaffold” equations
  1. One step (6<sup>th</sup> Grade)
  2. Two step (7<sup>th</sup> Grade)
  3. Distributive
  4. Variables on both sides
  5. Multistep Equations
- Ninth grade algebra starting with 6<sup>th</sup> grade, one step equations, then 7<sup>th</sup> grade, etc.
- Worse when compounded by extended beginning of the year review
- Why am I out of time?
- “Those students” get less access to grade level material
- Even if students taught procedurally remember for a procedural unit test, what happens when the prompts are not as procedural or the following year when students have forgotten?

**If you just finished multistep expressions, then why not start there with equations to reinforce what you have just done? It allows you to get those kids who were almost there all the way there too.**

# Scaffolding Definition

Instructional scaffolding is the support given during the learning process... to promote a deeper level of learning... These supports may include the following:

*resources [using pictorial math provides a resource to employ]*

*a compelling task [not a simplified task]*

*templates and guides [two, three, or four column notes]*

guidance on the development of **cognitive skills**

*[How are students thinking about what they are doing?]*

These supports are **gradually removed** as students develop autonomous learning strategies, thus **promoting their own cognitive, affective and psychomotor learning skills and knowledge**. Teachers help the students master a task or a concept by **providing support [not simplifying content]**. The support can take many forms such as outlines, recommended documents, storyboards, or key questions.

[https://en.wikipedia.org/wiki/Instructional\\_scaffolding](https://en.wikipedia.org/wiki/Instructional_scaffolding)

# Context as Support: A Compelling Task

**I went through the drive-thru and gave them \$10. I got back two hamburgers and \$4. How much did the burgers cost?**

**Students can do this problem without instruction in how to solve an equation, because the context is the scaffold. This refutes the argument that students must start with one step equations before solving two step equations.**

**Start with the conceptual, see how students do it, and then formalize a procedure based on what they did.**



# Equations: Four Column Notes

Fold Your Paper into Four Columns

Symbolic

Pictorial

What You Did

Property

*I don't have the space here to show all four columns so I will do the symbolic and pictorial only.*

# Equations

## Given & Distributive Property

Symbolic

Pictorial

$$2(-3x + 4) + 2x = -2x + 2$$



*Students usually do not know where to begin so I have them draw the shapes and draw the goal as the first step.*



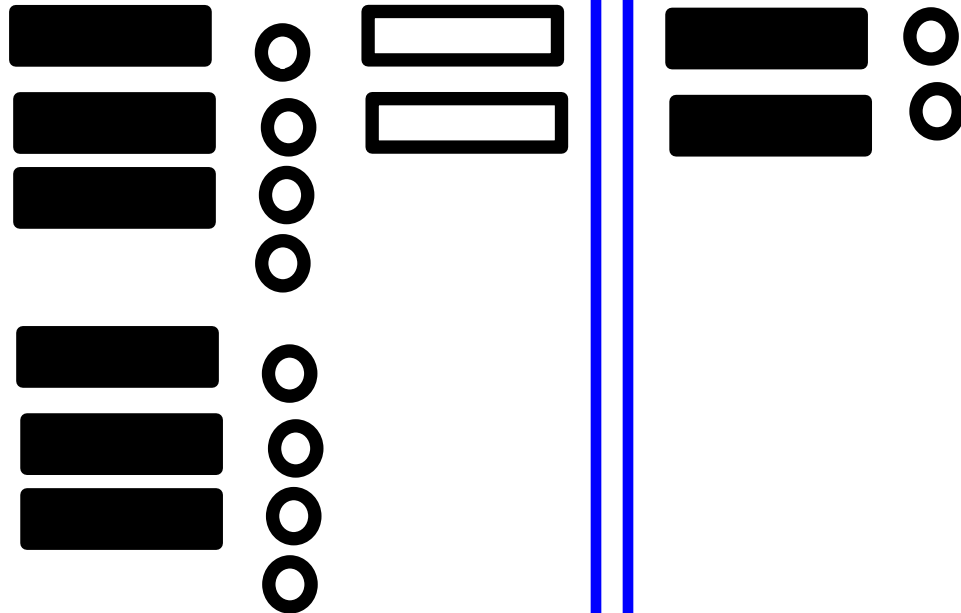
# Equations

## Given & Distributive Property

Symbolic

Pictorial

$$\begin{aligned} 2(-3x + 4) + 2x &= -2x + 2 \\ -6x + 8 + 2x &= -2x + 2 \end{aligned}$$



*Emphasize that you have to distribute to even draw the shapes.*

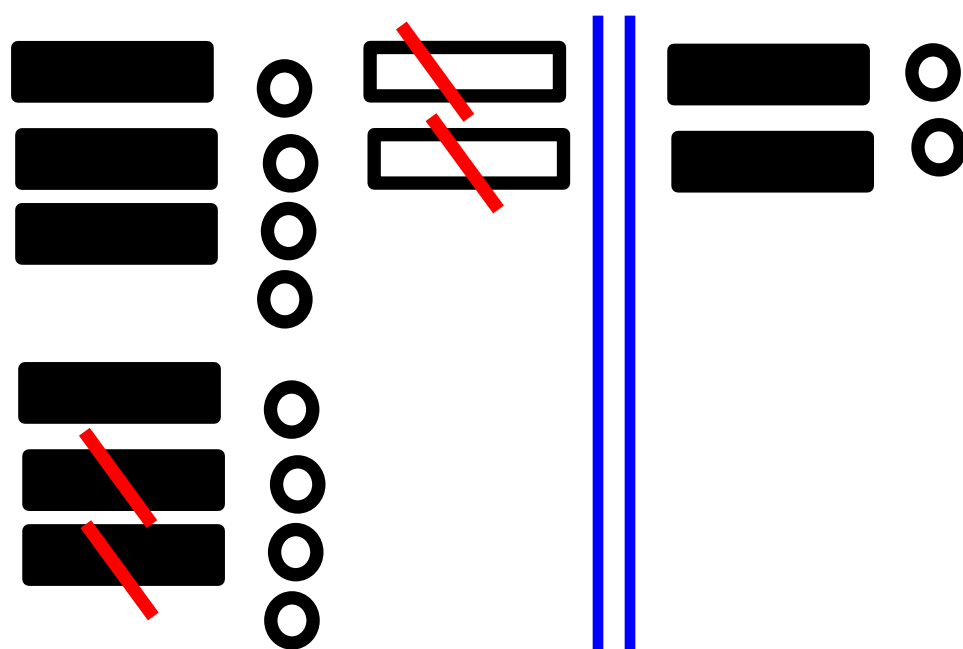
# Equations

## Combine Like Terms & Additive Inverse

Symbolic

Pictorial

$$\begin{aligned} 2(-3x + 4) + 2x &= -2x + 2 \\ -6x + 8 + 2x &= -2x + 2 \\ -4x + 8 &= -2x + 2 \end{aligned}$$



*Equations are just two equivalent expressions. Simplify each side.*

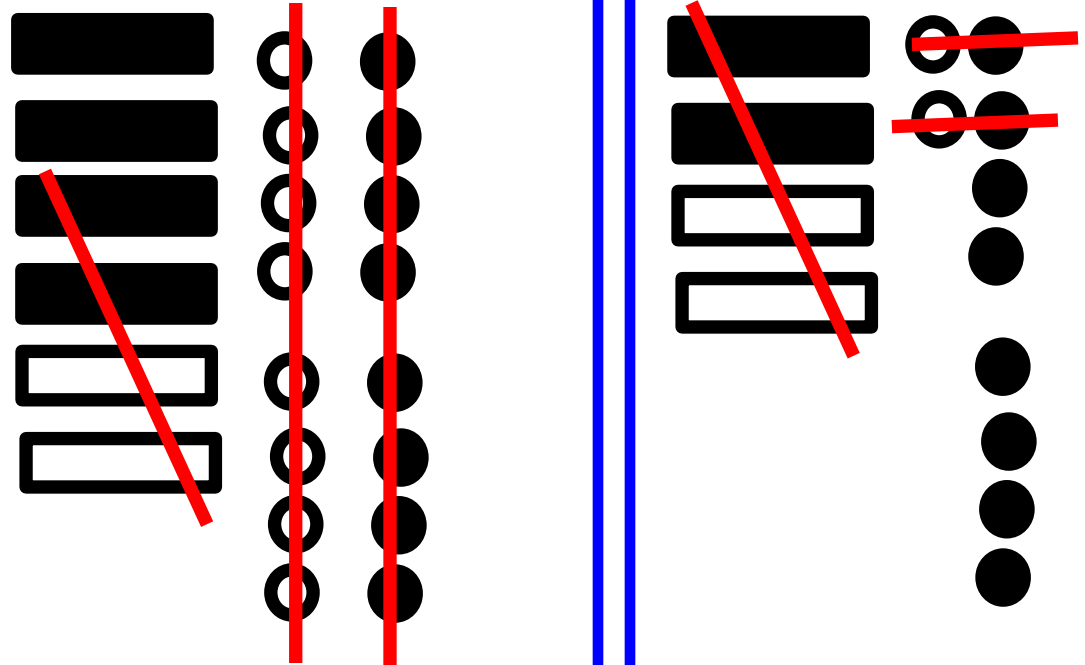
# Equations

## Addition Property of Equality

Symbolic

Pictorial

$$\begin{aligned}
 2(-3x + 4) + 2x &= -2x + 2 \\
 -6x + 8 + 2x &= -2x + 2 \\
 -4x + 8 &= -2x + 2 \\
 +2x - 8 &
 \end{aligned}$$



*What makes a zero pair? Variables on one side & constants on the other.*

*You can do this in two steps. Once they understand the goal though, they will be comfortable with the “double switch.”*

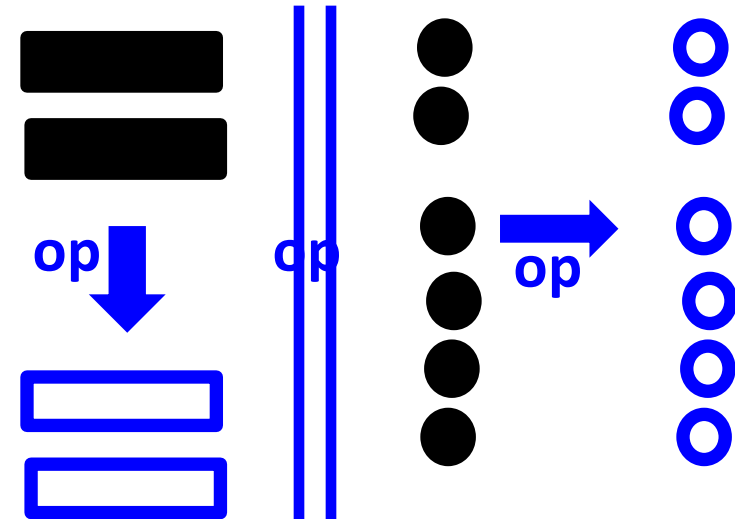
# Equations

## Multiplication Property of Equality

Symbolic

Pictorial

$$\begin{array}{rcl}
 2(-3x + 4) + 2x & = & -2x + 2 \\
 -6x + 8 + 2x & = & -2x + 2 \\
 -4x + 8 & = & -2x + 2 \\
 +2x - 8 & & +2x - 8 \\
 -2x(-1) & = & -6(-1) \\
 \text{op} & \text{op} & \text{op}
 \end{array}$$



*My goal is one positive unknown,  $x$ . I need the opposite of what I have so multiply both sides by  $-1$ . I have students write op-op-op, especially for knowing when to switch the signs in inequalities. Notice this gets rid of any double negatives division.*

# Equations

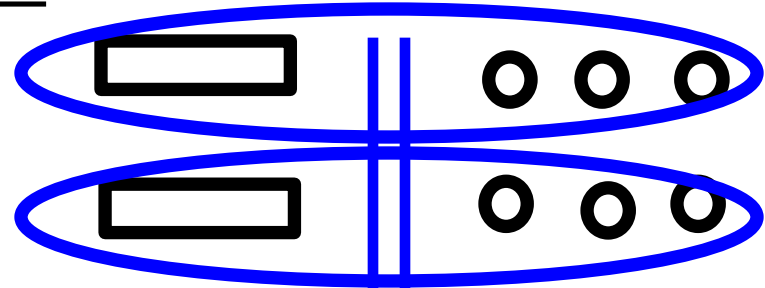
## Division Property of Equality & Solution

Symbolic

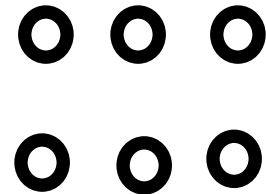
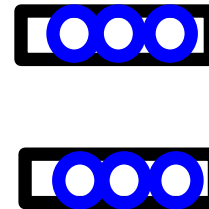
$$\begin{aligned}
 2(-3x + 4) + 2x &= -2x + 2 \\
 -6x + 8 + 2x &= -2x + 2 \\
 -4x + 8 &= -2x + 2 \\
 +2x - 8 &+2x - 8 \\
 -2x(-1) &= -6(-1) \\
 2x &= +6 \\
 \hline
 2 &2 \\
 x &= 3
 \end{aligned}$$



Pictorial



OR



*Make the coefficient one. If 2x contain 6, then how many in one? Did I subtract?  
What operation did I use? Divide by the coefficient to make a single x.*

# Next Level of Abstraction

## Drawing Around the Symbols

$$\begin{array}{l}
 2(-3x + 4) + 2x = -2x + 2 \\
 \boxed{-6x} \textcircled{+8} \boxed{+ 2x} = \boxed{-2x} \textcircled{+2} \\
 \boxed{-4x} \textcircled{+8} = \boxed{-2x} \textcircled{+2} \\
 \boxed{+2x} \textcircled{-8} \quad \quad \quad \textcircled{+2x} \textcircled{-8} \\
 \begin{array}{r}
 -2x(-1) \\
 \hline
 2x \\
 2 \\
 x
 \end{array}
 \end{array}$$

*Same team or  
do they fight?*

Where do I begin?

You cannot draw shapes around parenthesis so distribute first to get rid of them.

Now draw your shapes & your goal.

Put variables on the left and circles on the right.

Opposite Side,  
Opposite Sign.

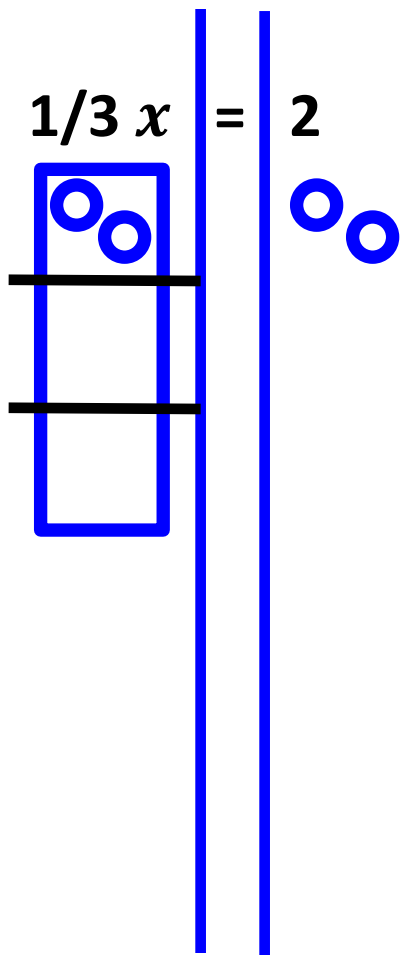
WAIT! Op-Op-Op  
(Multiply by  
Negative One)

Divide by the  
coefficient.

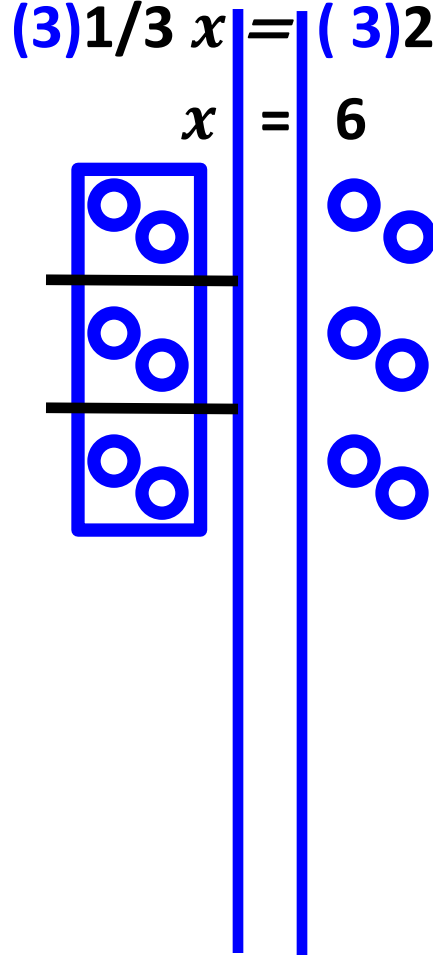
*This is a good time to introduce larger coefficients or even fractional coefficients if you need to remediate fractions or decimals.*

# Equations

## What About Unit Fractions?

$$\frac{1}{3}x = 2$$


SO...

$$(3)\frac{1}{3}x = (3)2$$
$$x = 6$$


You can push for understanding by illustrating simple fractions with students.

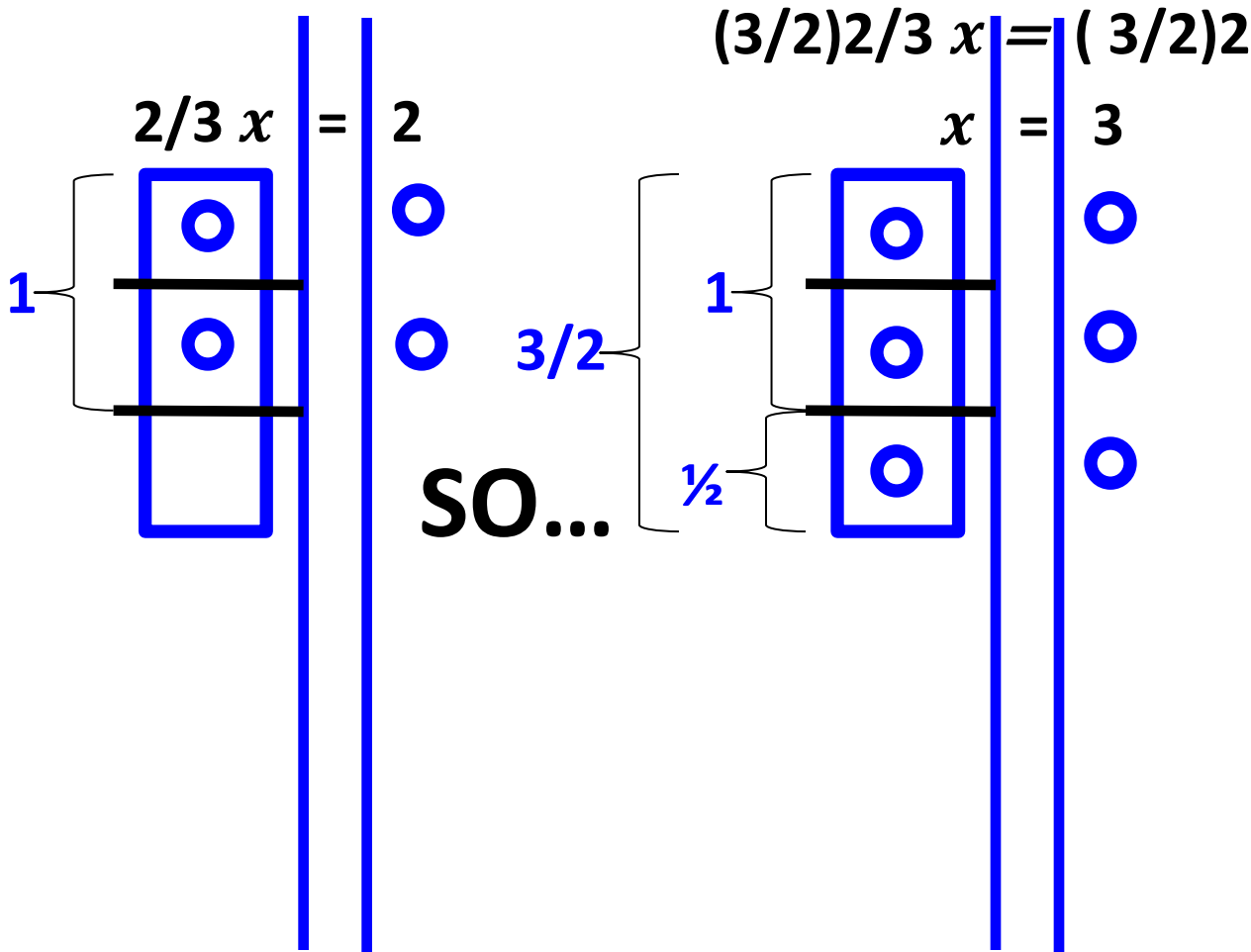
What did I have to do to change the coefficient of one third to one?

I multiplied by 3 on both sides  
(Multiplication Property of Equality). What about one fourth? One fifth? Etc.

*This can be extended to multiplying to get rid of fractions first.*

# Equations

## What About Fractions?



You can push for understanding by illustrating simple fractions with students.

What did I have to do to change the coefficient of two thirds to one?

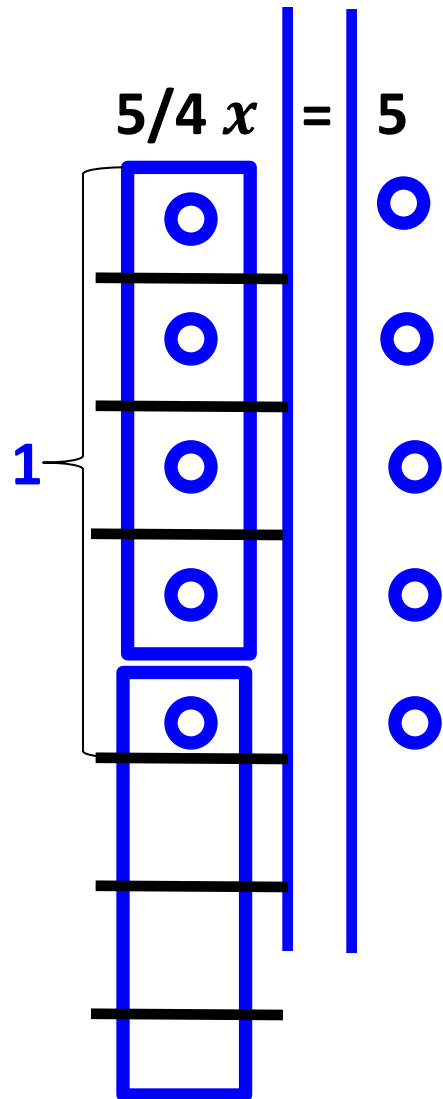
I multiplied by  $1\frac{1}{2}$  or  $\frac{3}{2}$  on both sides  
(Multiplication Property of Equality).

Does the reciprocal always work? What about improper fractions?

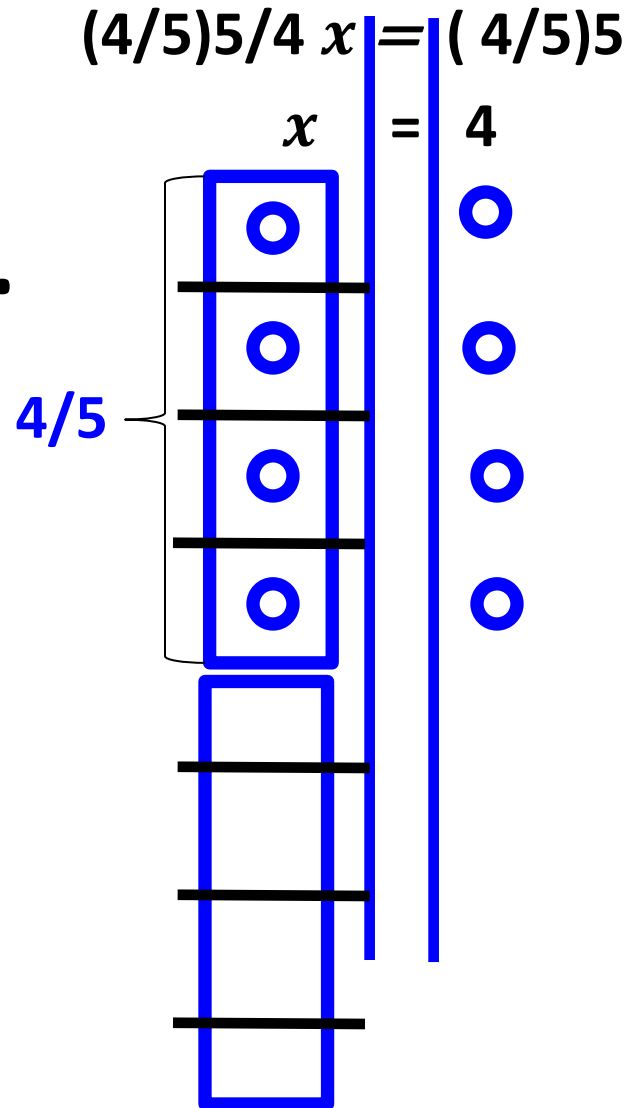


# Equations

## What About Improper Fractions?



SO...



What did I have to do to change the coefficient of five fourths to one?

I multiplied by  $4/5$  on both sides (Multiplication Property of Equality).

Would this be a good way to build some understanding of fractions without interrupting your teaching?

Reviewing in context

# Giving Students Procedures

## Should I or Shouldn't I?

Students should generalize to procedures themselves so they are more likely to remember them.

If you must provide them, then they should be a temporary resource for students. Keep them simple, short and always true:

1. Draw the shapes and the goal [this will help them begin]
2. If there are parenthesis, then you don't have a single group so you cannot draw the shapes. You must **distribute first, if necessary.**  $3(2x + 1)$  is not equivalent to  $32x + 1$ . **NOT**  $3(2x + 1)$
3. Simplify each equivalent expression, if possible.
4. Collect variables on one side & constants on the other.
5. Make the coefficient of the variable one.

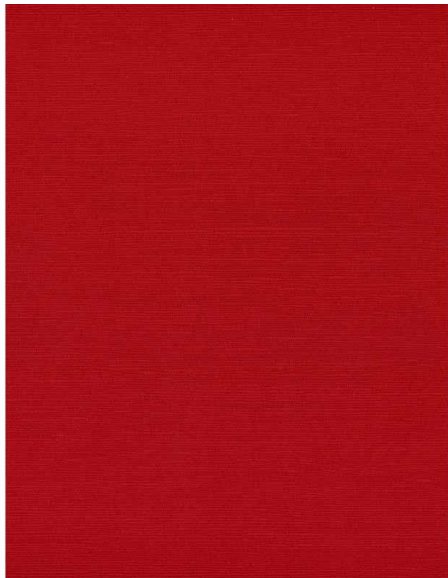
# What Materials Did I Use Today?

Two colors of cardstock

Slider bags

Folded blank paper

***Cheapest Manipulatives EVER!***

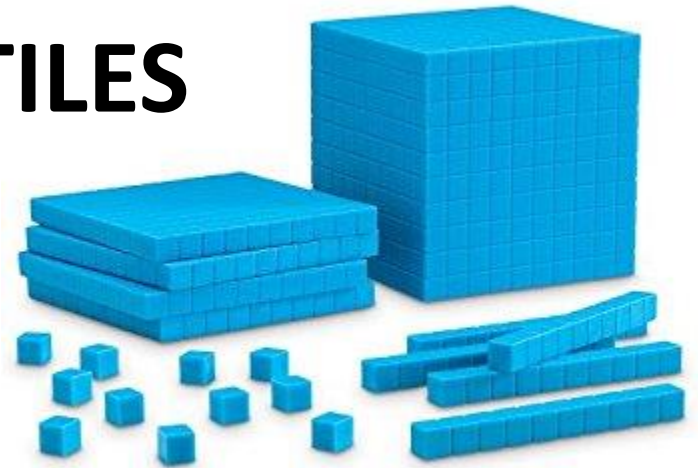


# Base 10 Blocks as **ALGEBRA TILES**

Some might argue against having the same manipulative as base ten and algebra tiles.

My counter argument is why keep two manipulatives when one will do?

More importantly, I *want* them to make connections between base ten and algebraic representations of any base to aid in understanding and retention.



# Base 10 Blocks as ALGEBRA TILES

Store bought algebra tiles have “unknown” side lengths as they are trying to reinforce the that quality of the variable.

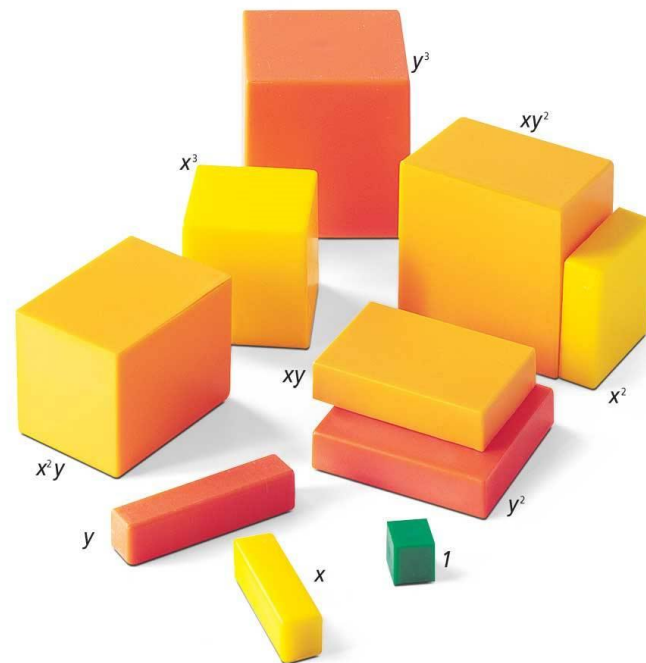
If you measure these tiles with the one, then you find they are usually between three and four. Does that mean it has a fractional side length?

All models have inherent flaws; choose the flaws that reinforce the learning when you can.



***I don't prefer the multiple color tiles. I prefer to use color to distinguish between positive and negative or different variables.***

# Algebra Tiles with Color as Different Variables or a Constant



*AlgeBlocks allow for work in three dimensions, cubic expressions & equations, and work with two variables,  $x$  &  $y$ .*

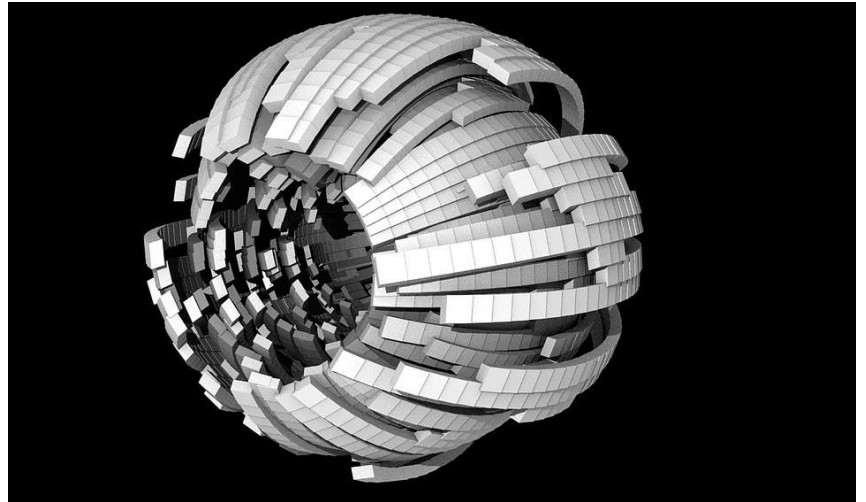
*<http://www.hand2mind.com/item/algeblocks-manipulative-starter-set/9241> Expensive @ \$200 though.*



# Where Can I Find It?

- This presentation will be posted on my fledgling website  
[www.meaningformemory.com](http://www.meaningformemory.com)
- Email me if you are interested in larger, precut consumable manipulatives -  
[David.Mattoon@gmail.com](mailto:David.Mattoon@gmail.com)
- COMING FALL 2018! Look for a [workbook](#) on how to implement this in your classroom along with premade activities for your students
- Please leave [feedback & questions](#) on the site.

Using Base Ten to See & Understand Algebra



Using Algebra to See & Understand Base Ten