Algebra as Area: The Distributive Property

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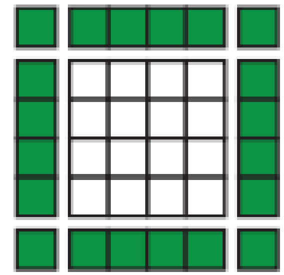
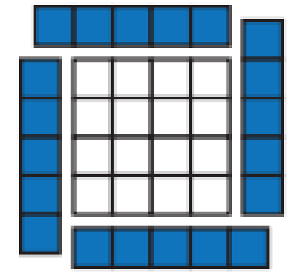
Distributive Property:
Inquiry

Inquiry Leading to the Distributive Property

3. In the well-known "Pool Border Problem," students are asked to determine the number of tiles needed to construct a border for a pool (or grid) of size $n \times n$, represented by the white tiles in the figure. Students may first examine several examples and organize their counting of the border tiles, after which they can be asked to develop an expression for the number of border tiles, B (MP.8). Many different expressions are correct, all of which are equivalent to $4n+4$. However, different expressions arise from different ways of seeing the construction of the border. A student who sees the border as four sides of length n plus four corners might develop the expression $4n+4$, while a student who sees the border as four sides of length $n+1$ may find the expression $4(n+1)$. It is important for students to see many different representations and understand that these representations express the same quantity in different ways (MP.7).



$B = 4n + 4$



$B = 4(n + 1)$

Adapted from NCDPI 2013b.

The Five Practices for Orchestrating a Math Discussion is a useful routine for debriefing this activity.

Use colored manipulatives to show student thinking, a small poster presentation explaining a student's/small group's method, follow with a gallery walk with the purpose of getting ideas to revise your own rough draft, then create a final draft.

Distributive Property: Context to Combat not Distributing to Both Terms

Context as Support

My friend and I visited a local fast food restaurant. We both ordered the same thing, and we both got the same change back. We each received a burger and four dollars. If *b* represents the cost of the burger, then write a single expression for the value of what we <u>both</u> received.

Context as Support

We each received a burger and four dollars.

$$1(b+4)+1(b+4)$$

$$2(b+4)$$

$$2b+8$$

What are the benefits of each expression above?

How could you go from the top expression to the middle expression? What mathematical steps would you take? Hint: Consider that $1x+1x=2x$

How could you go from the top expression to the bottom expression?

We each received a burger and four dollars.

$$1(b+4)+1(b+4)$$

$$1x+1x=2x \; where \; x=b+4, therefore:$$

$$2(b+4)$$

$$2(b)+2(4)$$

$$2b+8$$

What are the benefits of each expression below?

We each received a burger and four dollars.

$$1(b+4)+1(b+4)$$

$$2(b+4)$$

$$2b+8$$

What are the benefits of each expression above?

We each received a burger and four dollars.

$$1(b+4)+1(b+4)$$

You see each person's order.

$$2(b+4)$$

You see two of the same order.

$$2b+8$$

You see what they both got together.

Substitution into a Linear Expression

If the burgers cost six dollars, or $b = \$6$, then what was the value of what we both received together? Use any expression. Why did you choose the one you did?

$$(b + 4) + (b + 4)$$

$$2(b + 4)$$

$$2b + 8$$

Substitution into a Linear Expression

If the burgers cost six dollars, or $b = \$6$, then what was the value of what we both received together?

$(b+4)+(b+4)$	$2(b+4)$	$2b+8$
$[(6)+4]+[(6)+4]$	$2[(6)+4]$	$2(6)+8$
$[10]+[10]$	$2[10]$	$12+8$
20	20	20

The value of what we received together was twenty dollars.

Did it matter which expression we used?

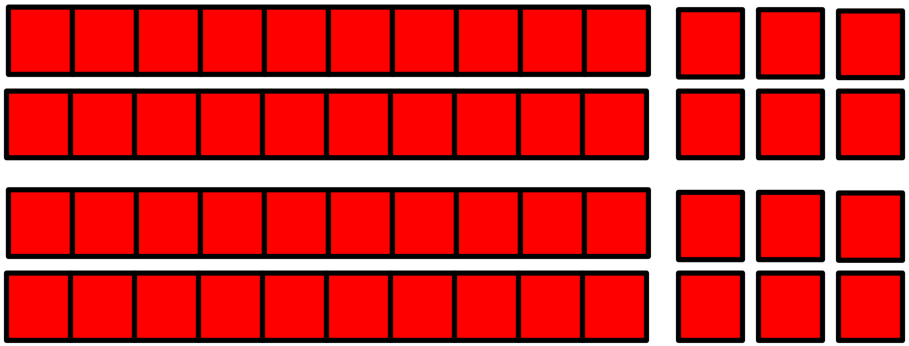
Why or why not?

This demonstrates that each of these expressions are <u>equivalent</u>.

Distributive Property: Mental Math

Distributive Property:
How do you see $2 \cdot 26$?

What is two times 26 or 26 two times?



How did you do it? How did you see it?

Who cares? When am I ever going to use this?

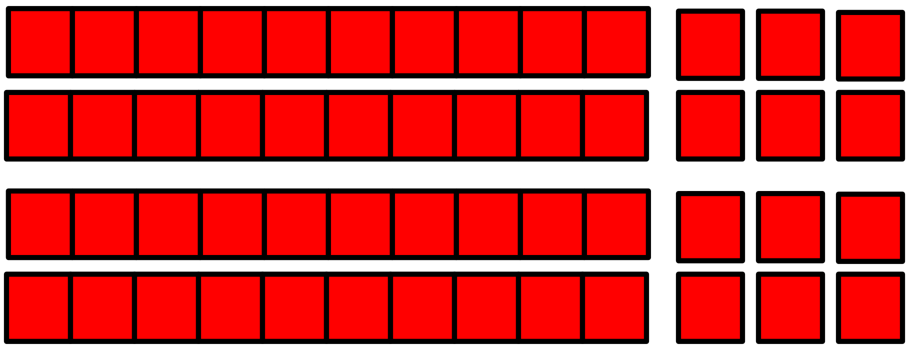
Most properties exist not only because they illustrate a universal mathematical truth, but because they are useful.

Try adding fractions with unlike denominators without using the Identity Property of Multiplication.

Distributive Property: How do you see $2 \cdot 26$?

Mattoon's MEANING 4 MEMORY

What is two times 26 or 26 two times?



How did you do it? How did you see it?

Did you add the four tens,

add the four threes,

and then add tens and ones together?

That is one way, but we are going to work with some others that involve multiplication.

Who cares? When am I ever going to use this?

Most properties exist not only because they illustrate a universal mathematical truth, but because they are useful.

Try adding fractions with unlike denominators without using the Identity Property of Multiplication.

Distributive Property: How do you see $2 \cdot 26$?

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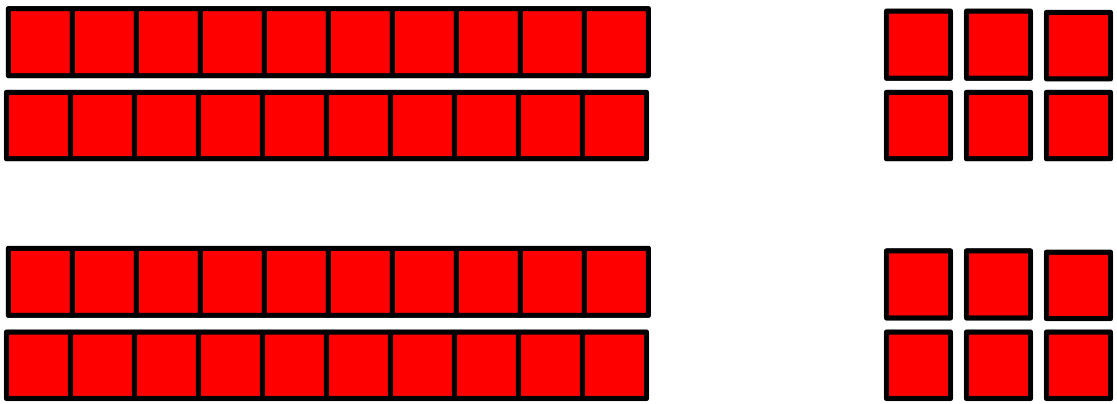

This would be

$$2(26) = 2(20) + 2(6) = 40 + 12 = 52$$

Who cares? When am I ever going to use this?

Most properties exist not only because they illustrate a universal mathematical truth, but because they are useful.

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Distributive Property: How do you see $2 \cdot 26$?

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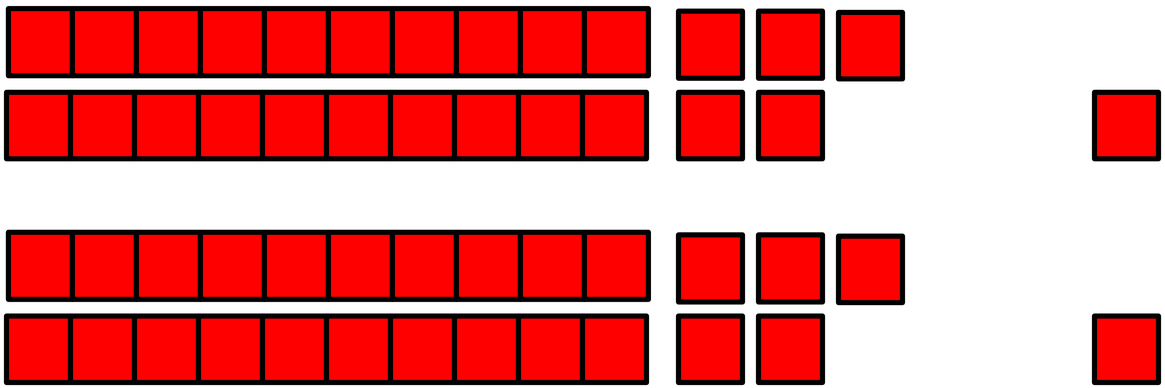


This would be

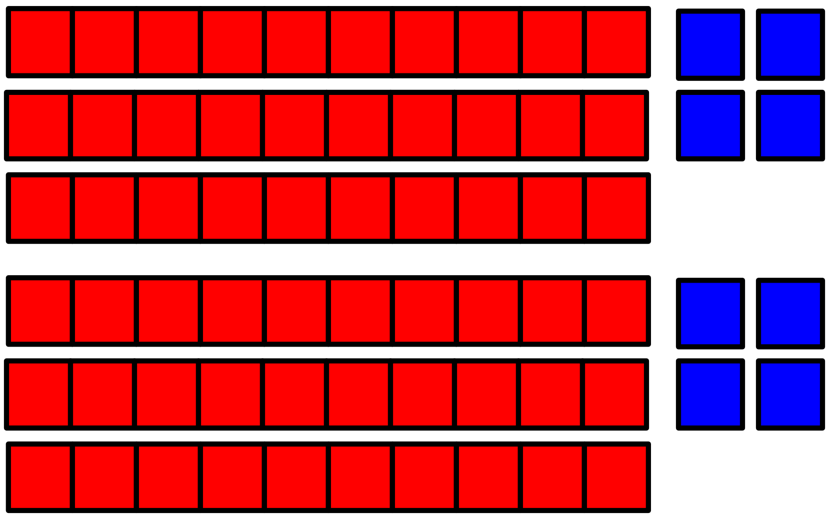
$$2(26) = 2(25) + 2(1) = 50 + 2 = 52$$

When using mental math, this might be a possibility if you know your 25 math facts or can think of it as 26 cents being a quarter and a penny

Distributive Property: How do you see $2 \cdot 26$?

Mattoon's MEANING 4 MEMORY

What is two times 26 or 26 two times?



This would be

$$2(26) = 2(30) + 2(-4) = 60 - 8 = 52$$

What about negatives, represented here by the blue ones?

You might not do this for 26, but would it be a good strategy for 2·29 ? Who wants to draw nine little squares?

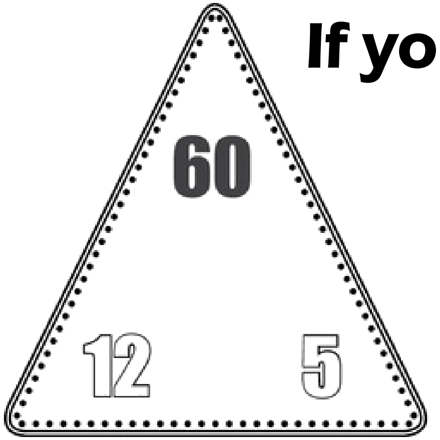
Distributive Property & Math Facts

Not sure what $5 \cdot 12$ is?

Use the distributive property!

This would be

$$5(12) = 5(10 + 2) = 5(10) + 5(2)$$
$$= 50 + 10 = 60$$



If you are going to use flash cards, then use the triangular cards. They do division too!

Not sure what $5 \cdot 12$ is? Notice 12 has two digits so split it into two parts.

First, decompose 12 (split it apart) into 10 and 2.

Multiply each smaller term by 5 rather than the larger term, 12.

Add the partial products.

Do it enough and your brain will memorize the math fact with no extra practice or flashcards.

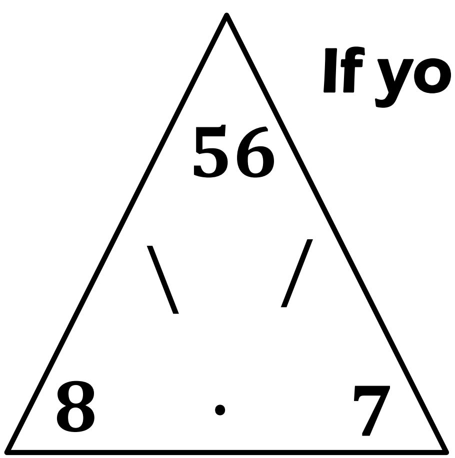
Distributive Property & Math Facts

Not sure what 8 · 7 is?

Use the distributive property!

This would be

$$8(7) = 8(5+2) = 8(5) + 8(2) = 40 + 16 = 56$$



If you are going to use flash cards, then use the triangular cards. They do division too!

What if the math fact are single digits?

First, decompose 7 (split it apart) into 5and 2.

Multiply each smaller term by 8 rather than the larger term, 7, by 8.

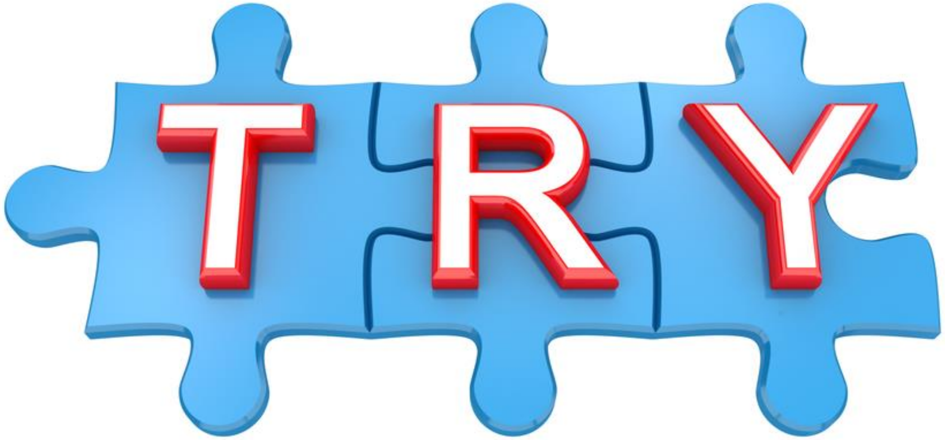
Add the partial products.

Do it enough and your brain will memorize the math fact with no extra practice or flashcards.

Distributive Property & Math Facts

Not sure what 5 ·143 is?

Use the distributive property!



What if one term has three digits?

First, decompose 143 (split it apart) into 100, 40 and 3. Multiply each smaller term by 5 rather than the larger term, 143, by 5.

Add the partial products.

Do it enough and your brain will memorize the math fact with no extra practice or flashcards.

Distributive Property & Math Facts

Mattoon's MEANING 4 MEMORY

Not sure what 5 ·143 is?

Use the distributive property!

This would be

$$5(143) = 5(100 + 40 + 3)$$
$$= 5(100) + 5(40) + 5(3)$$
$$= 500 + 200 + 15$$
$$= 715$$

First, decompose 143 (split it apart) into 100, 40 and 3.

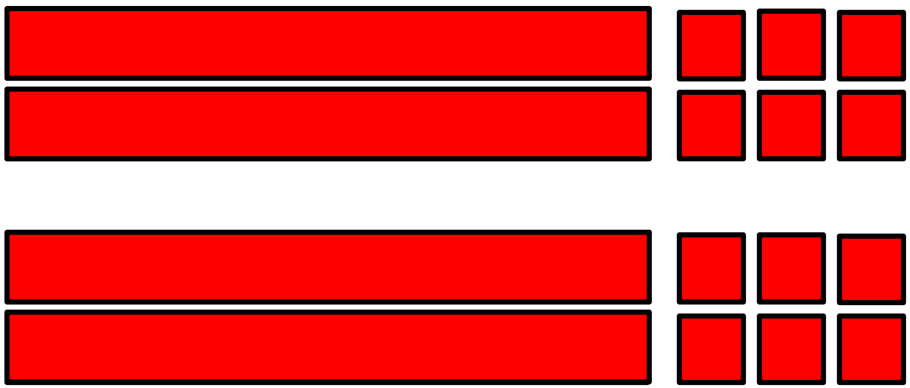
Multiply each smaller term by 5 rather than the larger term, 143, by 5.

Watch out for 5(40) as 5·4 creates its own zero in 20 plus the other zero from the ten in 40.

Add the partial products.

Distributive Property: Turn the linear terms over

What is two times 2x + 6 or 2x + 6 two times?



How did you do it? How did you see it?

Turn the linear terms, the tens, over.

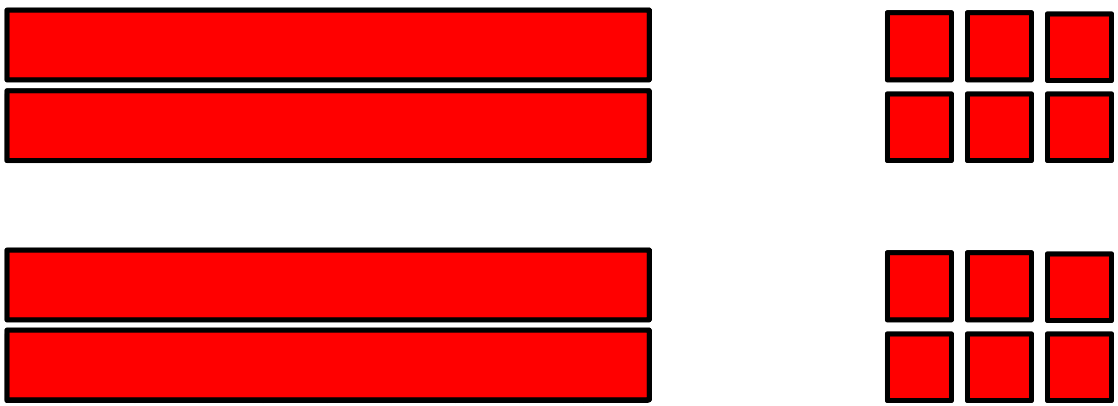
No lines of division mean we now dealing with any base (even though it still looks like base ten.)

You can no longer regroup as you do not know what base it is.

This lack of regrouping makes algebra easier than base ten when using tiles!

Distributive Property: Turn the linear tiles over

What is two times 2x + 6 or 2x + 6 two times?



This would be

$$2(2x+6)=2(2x)+2(6)=4x+12$$

You cannot combine 4x & +12 as they are not like terms, not the same shape.

You cannot get a finite numeric answer either unless you know what the base, x, is.

x could any number or base; it does not have to be ten.

What would the product be in base two, x=2?

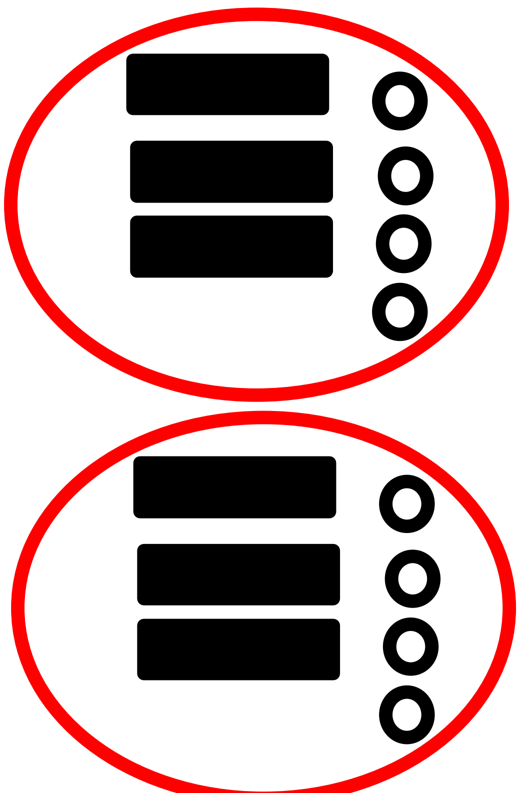
Base 5, x=5?

Distributive Property:
Grouping

Distributive Property: Reading & Drawing the Tiles

Mattoon's MEANING 4 MEMORY

$2(-3x + 4)$ Read, "Two groups of negative three x plus four or two groups of negative three times a number plus four." Drawn as:



One group

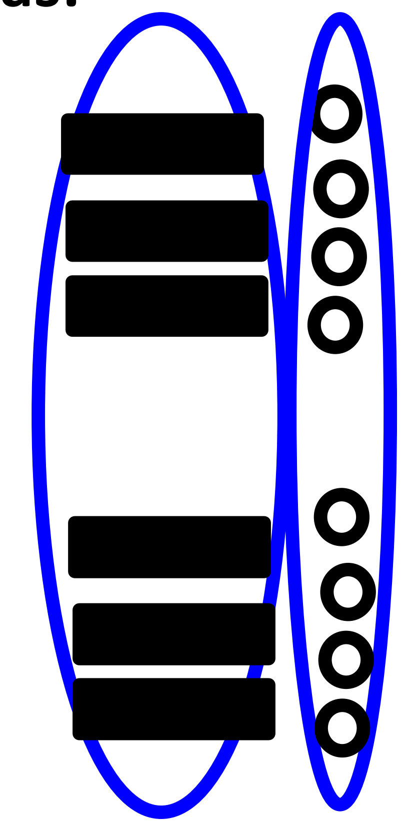
Two groups

How many negative x's?

How many constants?

So, the product is?

$-6x + 8$



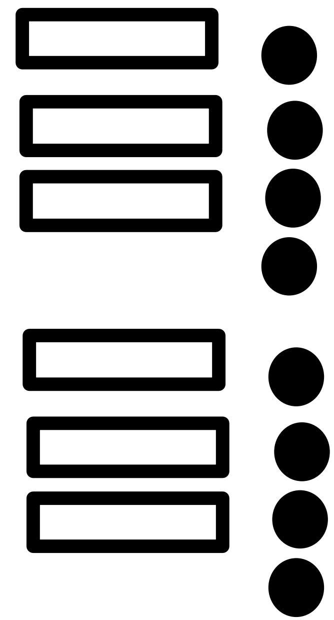
Most students haven't had the conceptual instruction for multiplication.

Most can multiply, but some do not realize it is making groups.

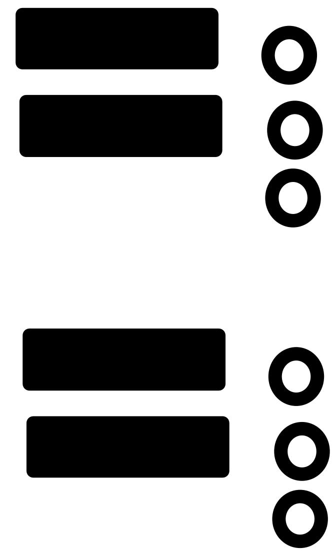
Others can tell you it is making groups, but it still does not inform their mathematics.

For example, they don't make groups to simplify. This needs to be

Distributive Property: Four Column Notes

Symbolic	Meaning in Words	Pictorial	Simplified
$-2(-3x + 4)$	The opposite of two groups of negative three times a number plus four		$6x - 8$
	Three groups of two times a number minus five		

Distributive Property: Four Column Notes

Symbolic	Meaning in Words	Pictorial	Simplified
			
$-2(4x-3)$			

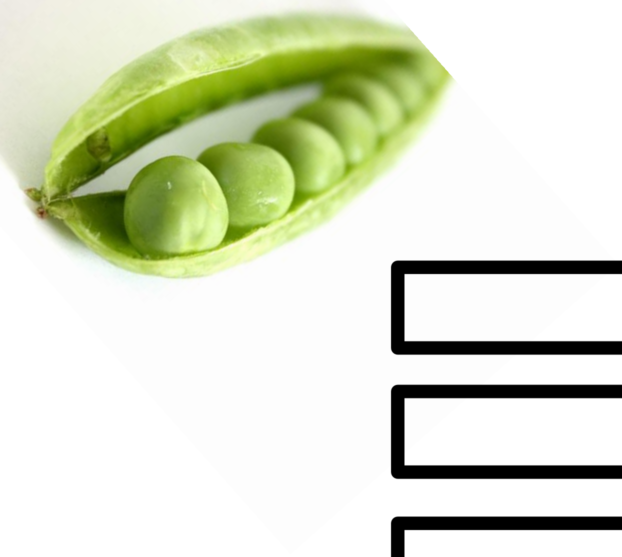
Linear Substitution:
Give Peas a Chance!

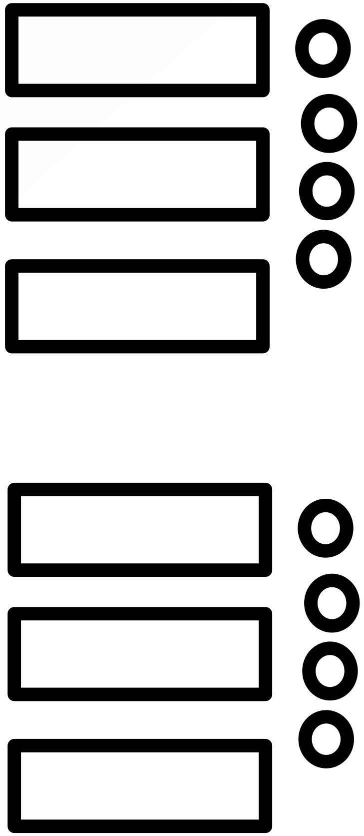
Another Context for Linear Substitution

- How are the peas arranged in a pod?
- You can think of a pea pod as a linear term.
- Let □ represent the peapod.
- Let ○ represent the peas

If p= # of peas in a pod, then…

Another Context for Substitution



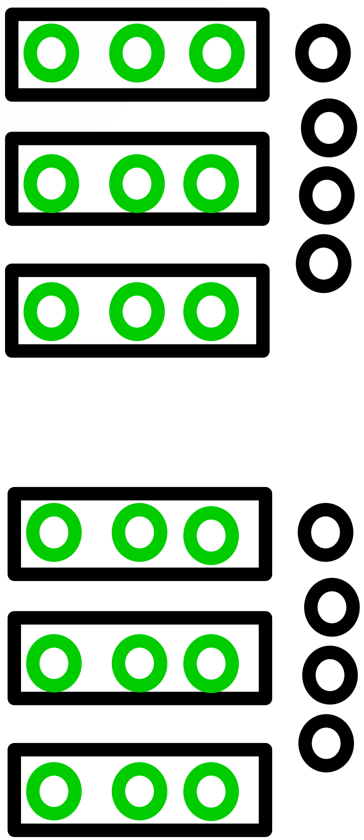


If p= # of peas in a pod, then..

How many total peas if the pea pods hold three peas each?

$2(3p + 4)$

Another Context for Substitution



If p= # of peas in a pod, then..

How many total peas if the pea pods hold three peas each?

$2(3p + 4)$

$6p + 8$

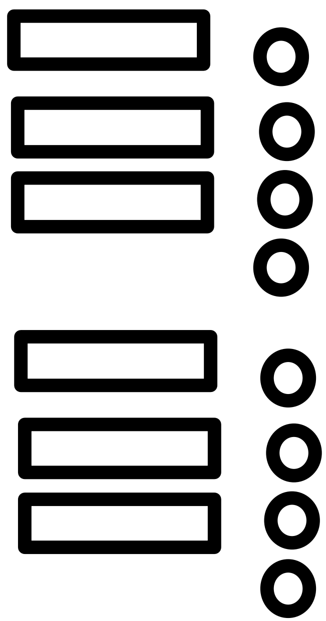
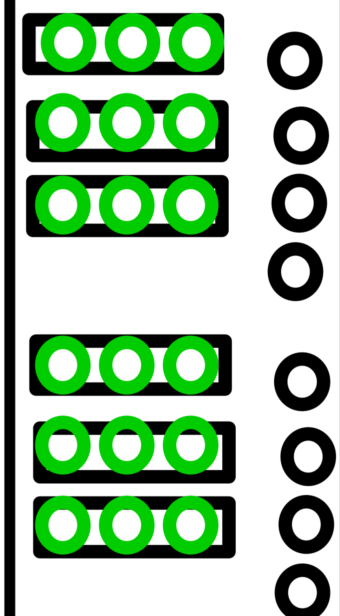
$6(3) + 8$

$18 + 8$

26

Linear Expressions: Five Column Notes

Fold Your Legal Paper into Five Columns

Linguistic in Context of Peas	Pictorial Expression	Symbolic Expression	If $p = 3$ Pictorial	If $p = 3$ Symbolic
Three pea pods and four extra peas, twice		$(3p + 4)2$		$(3p + 4)2$ $[3(3) + 4]2$ $[9 + 4]2$ $[13]2$ 26

Linear Expressions: Five Column Notes

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Linear Expressions: Five Column Notes

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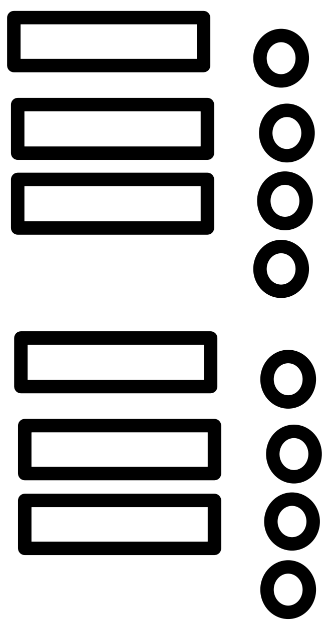

Fold Your Legal Paper into Five Columns

Linguistic in Context of Peas	Pictorial Expression	Symbolic Expression	If $p = 3$ Pictorial	If $p = 3$ Symbolic
		$(3p + 4)2$		

Linear Expressions: Five Column Notes

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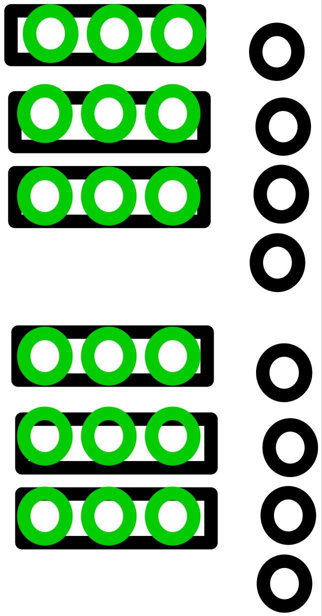
				

Fold Your Legal Paper into Five Columns

<u>Linguistic in Context of Peas</u>	<u>Pictorial Expression</u>	<u>Symbolic Expression</u>	<u>If $p = 3$ Pictorial</u>	<u>If $p = 3$ Symbolic</u>
				$[3(3) + 4]2$ $[9 + 4]2$ $[13]2$ 26

Context to Combat not following Order of Operations

Apple Picking for Order of Operations

Mario's family went to pick apples. Mario carried five apples in his t-shirt and his parents each carried a full basket with an equal amount of apples in them plus his parents each carried three apples in their free hand.

Draw a picture and write an expression for the situation

on your Frayer Model focus notes.

The amount of apples in the baskets are unknown

so you can draw empty baskets for now.

Apple Picking Frayer Model

Mattoon's MEANING 4 MEMORY

Mario's family went to pick apples. Mario carried five apples in his t-shirt and his parents each carried a basket with an equal amount of apples in them plus his parents each carried three apples in their free hand.

Picture

Apple Expressions

Name

Date

Per.

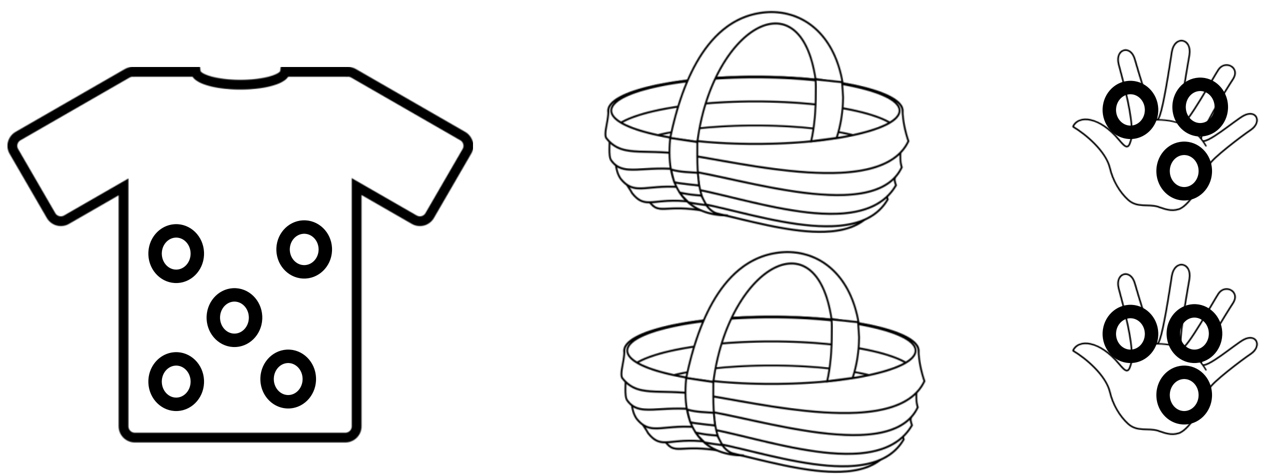
Substitution

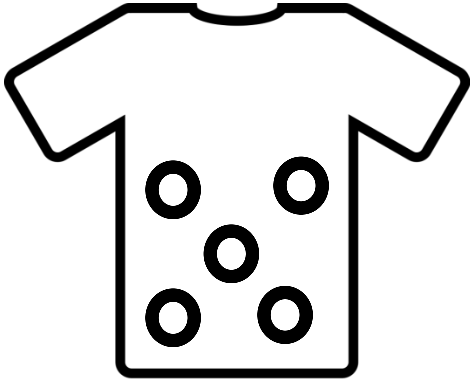
Symbolic Notation

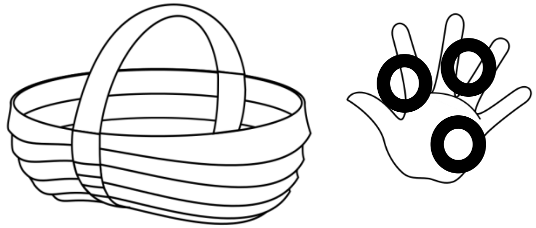
Apple Picking for Order of Operations

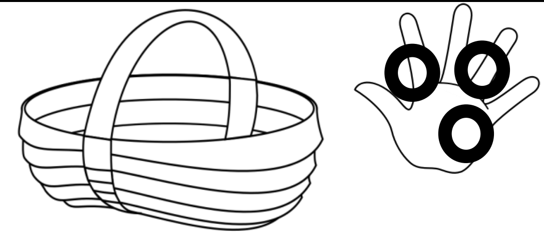
Mario's family went to pick apples. Mario carried five apples in his t-shirt and his parents each carried a basket with an equal amount of apples in them plus his parents each carried three apples in their free hand.

Draw a picture and write an expression for the situation.





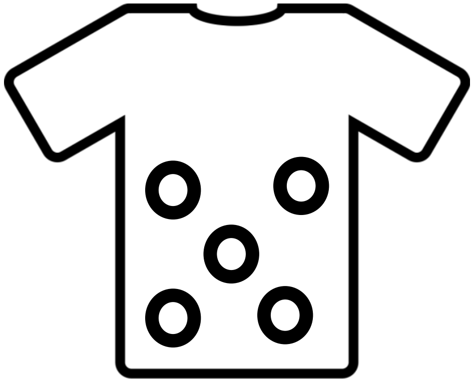


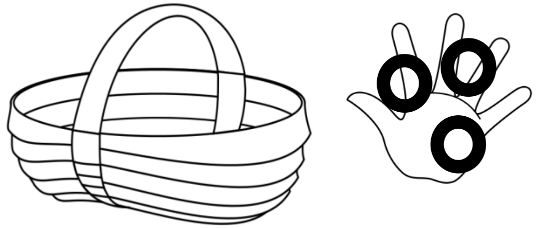


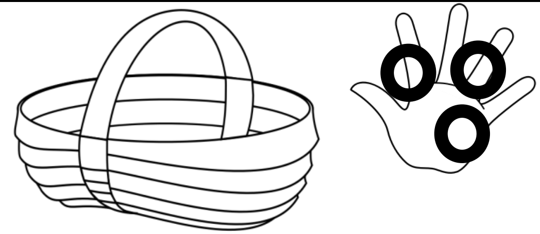
$$(5) + (b + 3) + (b + 3)$$

What do the parenthesis indicate?

Meaning & the Invisible One







$$(5) + (b + 3) + (b + 3)$$

Let's use the "invisible one".

How is $1(b + 3) + 1(b + 3) = 2(b + 3)$,

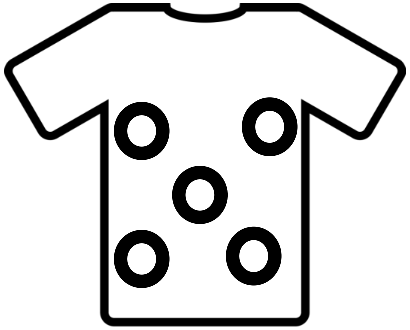
similar to $1x + 1x = 2x$?

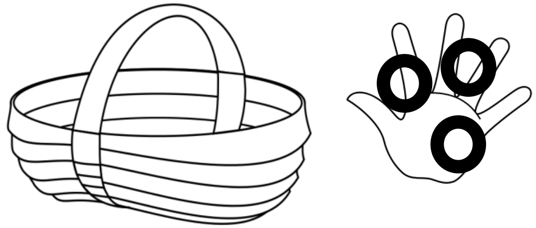
So, $(5) + 1(b + 3) + 1(b + 3)$ can be written as

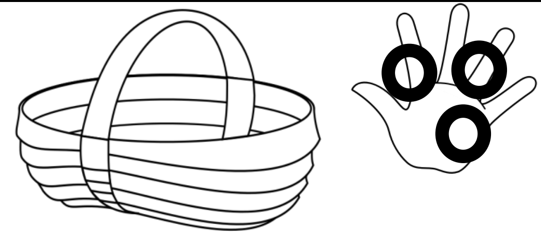
$$(5) + 2(b + 3)$$

What does the 2 mean?

Meaning & the Invisible One







Let's draw in the "invisible" one again. $1(5)+2(b+3)$

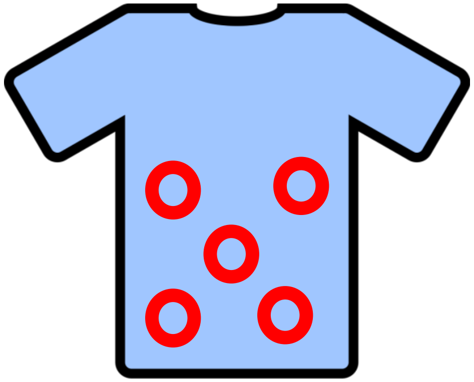
What does the five mean? What does the one mean?

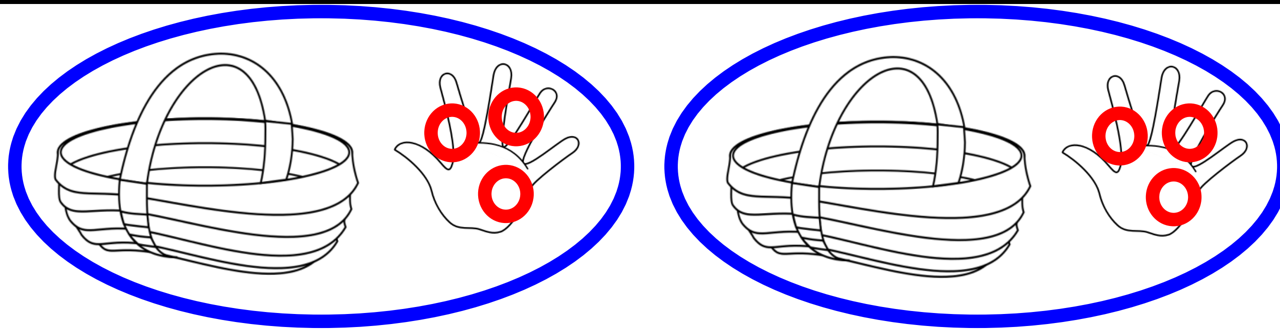
$1(5)+2(b+3)$ So, what do the quantities inside the parenthesis mean?

$1(5)+2(b+3)$ What do the quantities outside the parenthesis indicate?

Why can't you add the 5 and the 2 first in this expression? $5+2(b+3)$

Why Order of Operations Makes Sense





$$1(5) + 2(b + 3)$$

The quantities inside the parenthesis are the number of apples.

The quantities outside the parenthesis are the number of people.

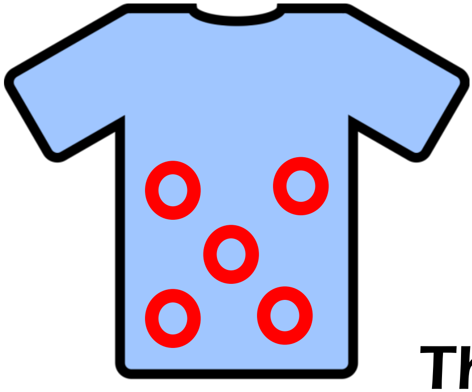
Why can't you add the 5 and the 2 first in this expression, $5 + 2(b + 3)$?

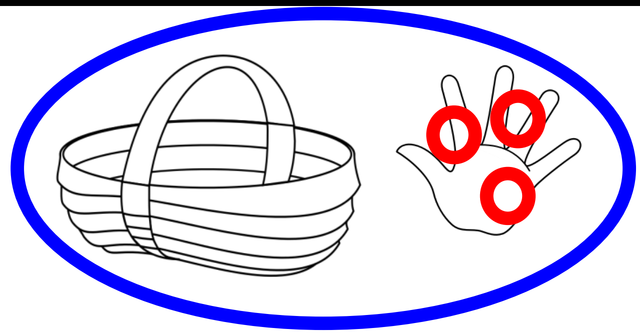
They are totally different quantities, five is the number of apples whereas two is a number of people or the number of groups!

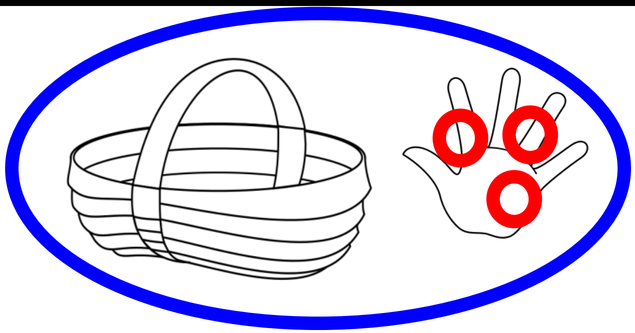
You must first use the number of groups to determine how many apples, and then you can add apples with apples.

Order of operations makes sense in context!

Why Order of Operations Makes Sense







There are 22 apples in each basket.

Find the total number of apples the Billy and his parents took home altogether. Substitute into each expression to demonstrate they are equivalent.

$$5 + 2(b + 3)$$

$$5 + 2b + 2(3)$$

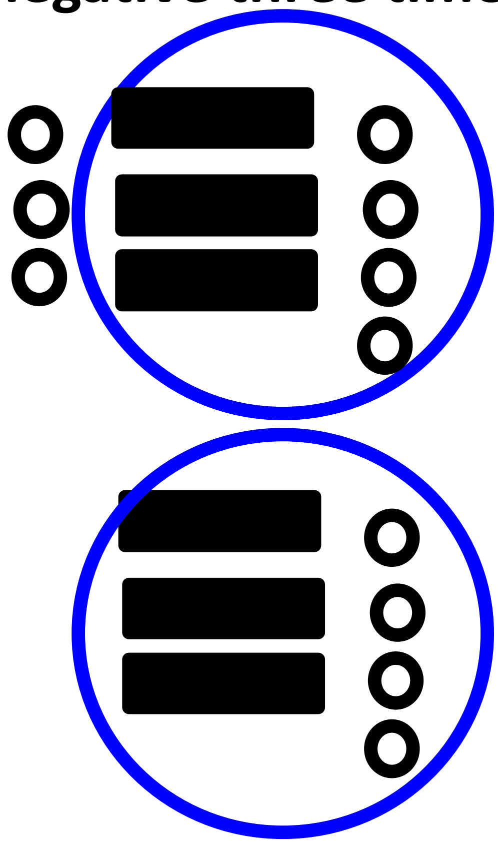
$$5 + 2b + 6$$

$$2b + 11$$

Record on your Frayer Model focus notes.

Distributive Property & Order of Operations

$3 + 2(-3x + 4)$ Read, "Three and two groups of negative three x plus four or three and two groups of negative three times a number plus four." Drawn as:



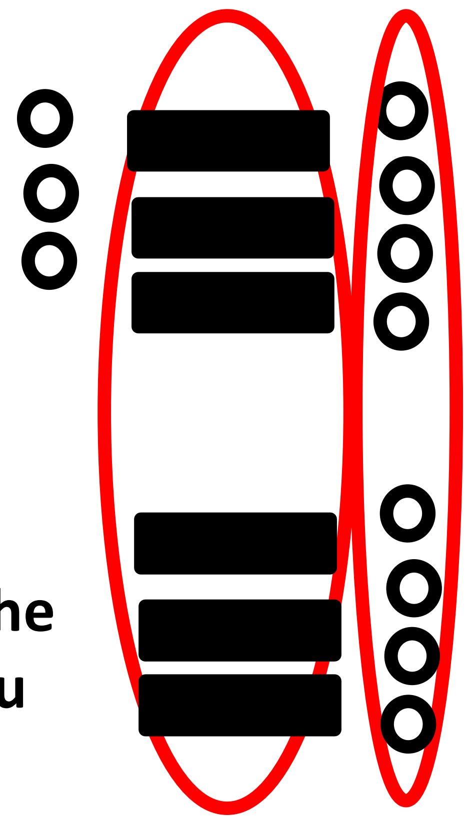
One group

Two groups

How many negative x's?

How many constants?

Remember the constants you started with!

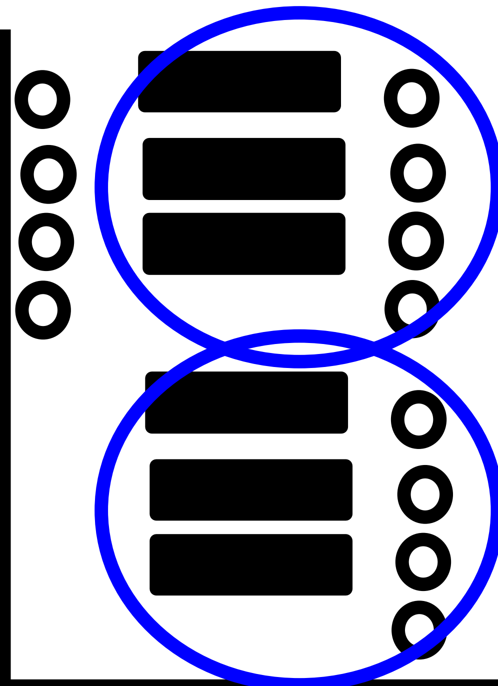


Like all of us, students tend to tackle what they understand best first; hence, the common error of $3 + 2(-3x + 4)$

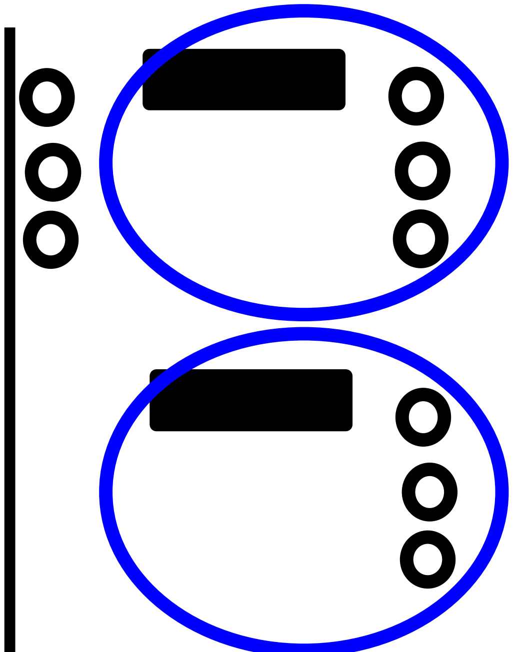
$5(-3x + 4)$.

Remind them of the apple lesson when they do so. Focus on the two being the number of groups, which is a totally different quantity.

Distributive Property: Order of Operations

Symbolic	Meaning in Words	Pictorial	Simplified
4 + 2(-3x+ 4)	Four and two groups of negative three times a number plus four		$-6x + 12$
	The difference between six and three groups of two times a number minus five		

Distributive Property: Order of Operations

Symbolic	Meaning in Words	Pictorial	Simplified
			
6 - 2(-3x+ 4)			

Distributive Property as Area: Final Level Abstraction

Multiply each term within the parenthesis by the number of groups.
Remember to take the sign when you do!

$$7 - 5(2x - 3)$$

$$7 - 10x + 15$$

$$-10x + 23$$

After doing all this work, students should begin asking if they have to draw the shapes.

If not, then ask them if there is a way to go from

$$5(2x + 3)$$

to

$$10x + 15$$

without drawing the shapes.

Distributive Property as Area: Final Level Abstraction

Multiply each term within the parenthesis by the number of groups. Remember to take the sign when you do!

$$-7\frac{3}{4}-\frac{7}{2}(2x-3)$$

$$-7\frac{3}{4}-7x+\frac{21}{2}$$

$$-7x+2\frac{3}{4}$$

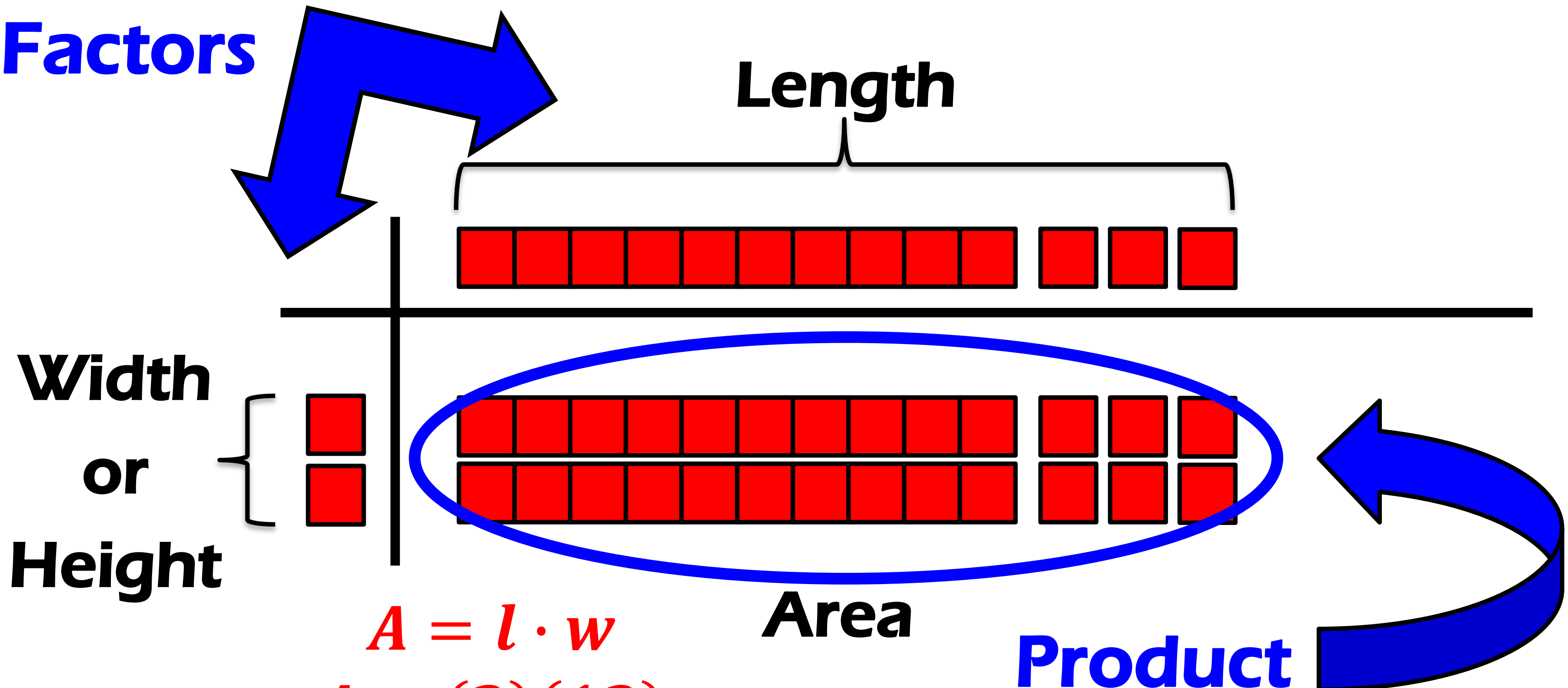
Sixth grade works with whole numbers on distributing and factoring linear terms. The difference in seventh grade is that it includes fractions.

This problem raises the ask to a seventh grade level for sure. What skills would you need to solve this single problem?

Less can be more.

Distributive Property:
Area

Distributive Property as Area: 2·13



$$A = l \cdot w$$

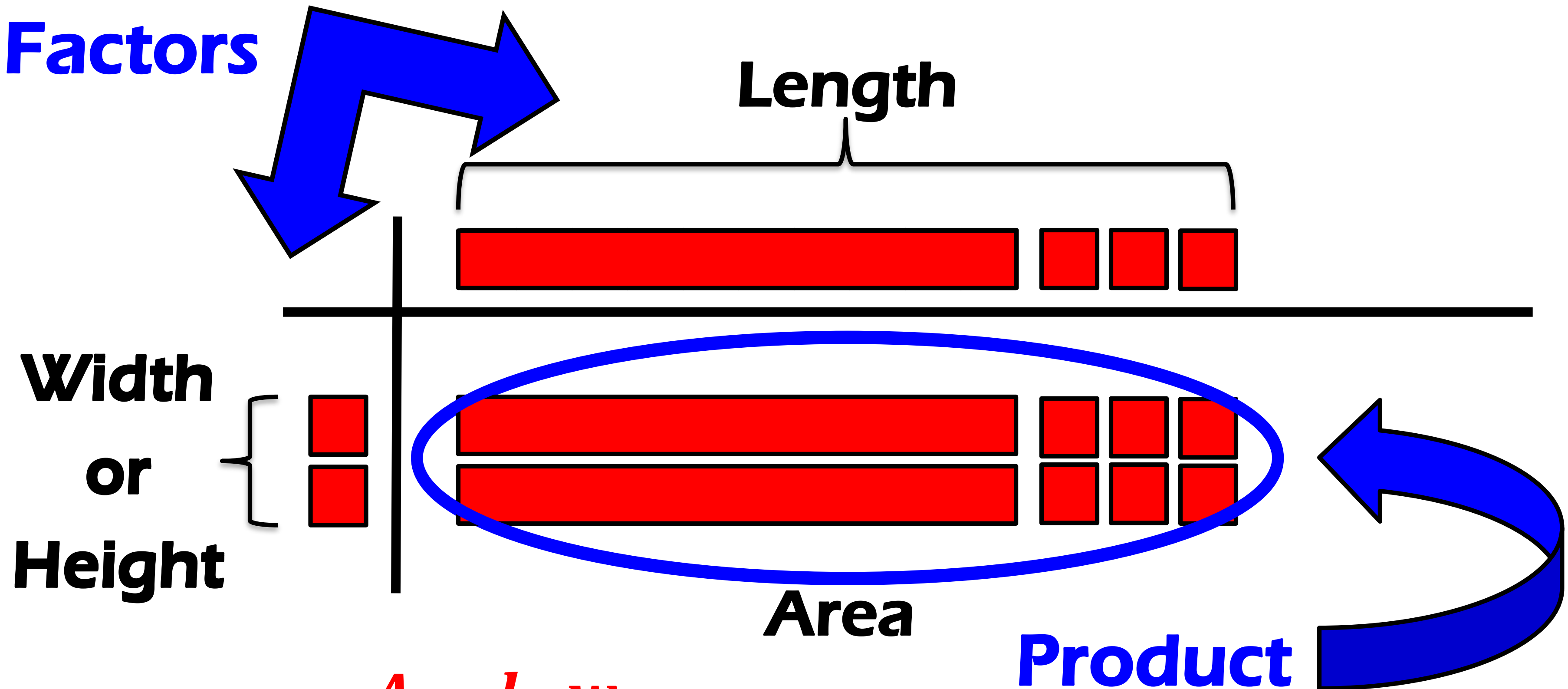
$$A = (2)(13)$$

$$A = 2(10) + 2(3)$$

$$A = 20 + 6$$

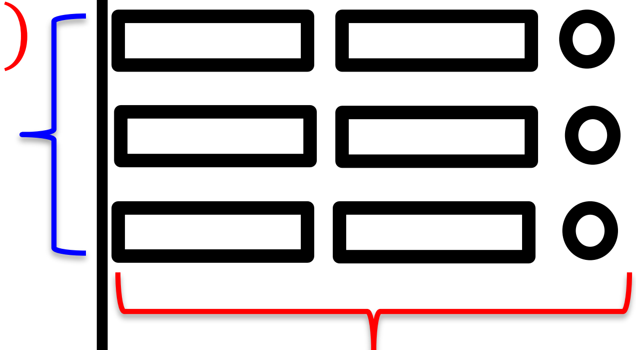
$$A = 26 \text{ square units}$$

Distributive Property as Area: 2·(x+3) Turn the Linear Terms Over



$$A = l \cdot w$$
$$A = (2)(x+3)$$
$$A = 2(x) + 2(3)$$
$$A = 2x + 6$$

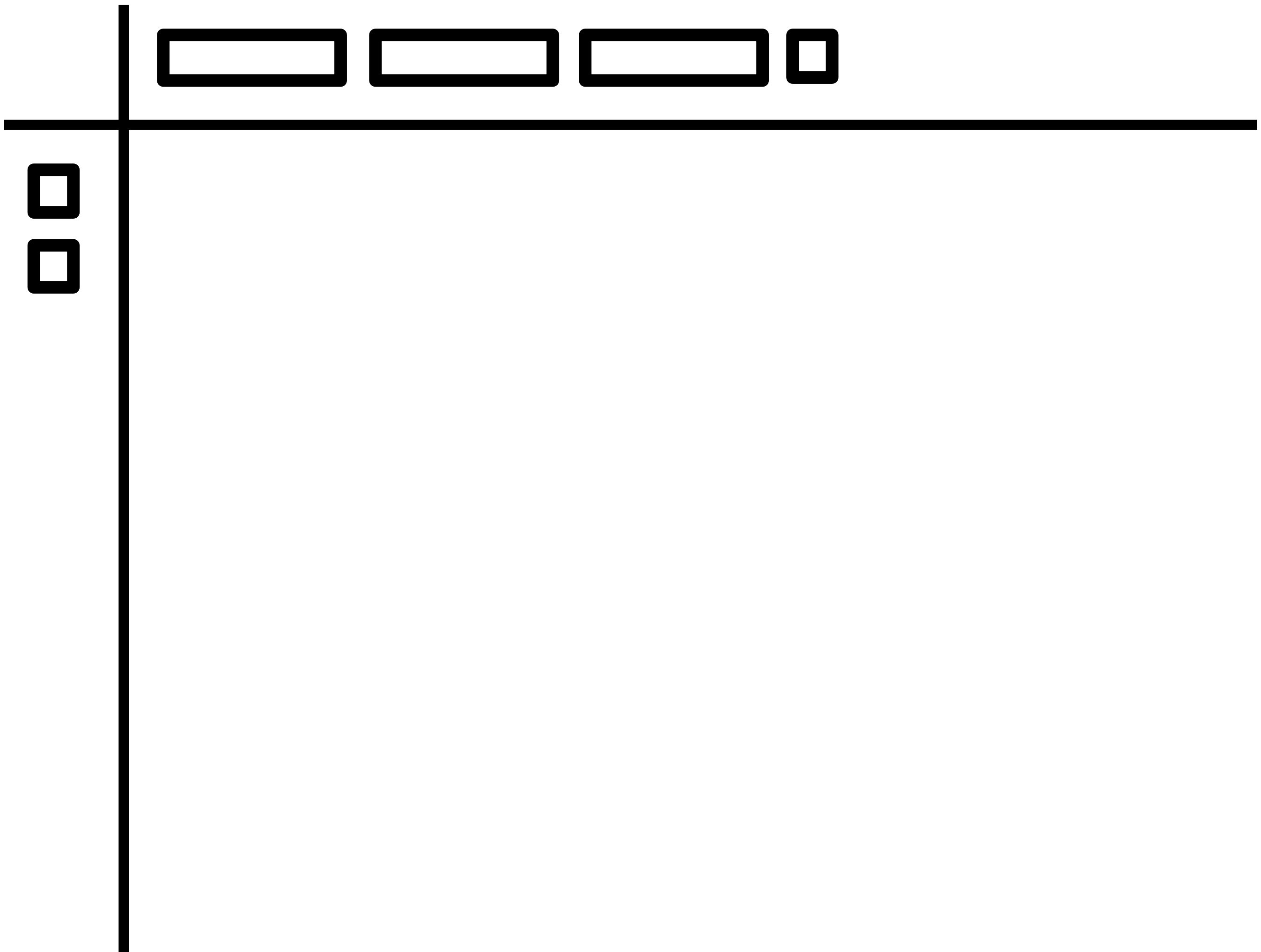
Distributive Property: Four Column Notes

Symbolic Expression	Pictorial Expression	Expanded Notation	Simplified Expression
$3(2x+1)$		$3(2x)+3(1)$	$6x+3$

Why aren't we drawing the factors?

They are the length of the sides so they are already there.

Distributive Property as Area: Whiteboards $2(3x + 1)$

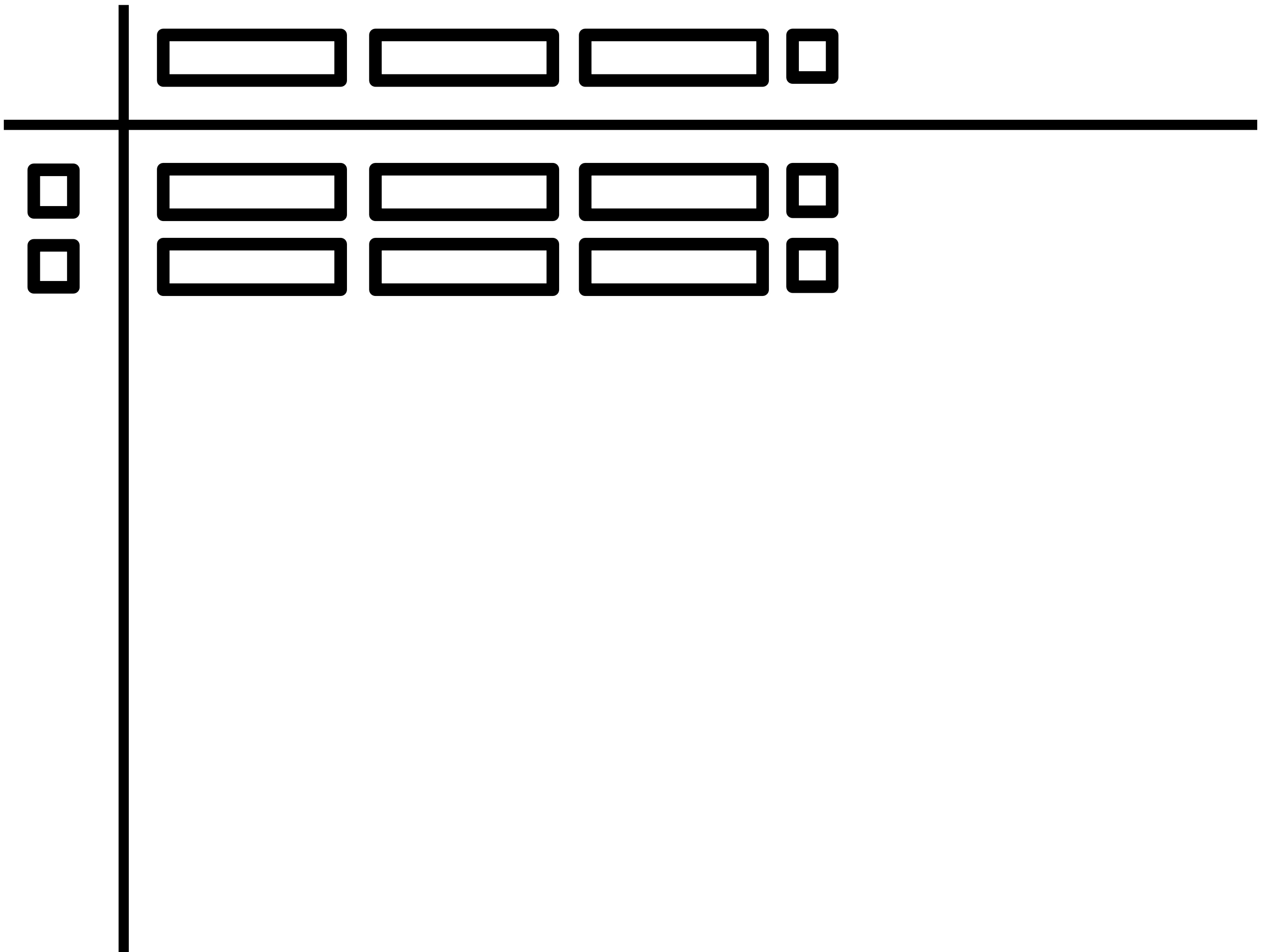


On your whiteboards, place tiles or draw $2(3x + 1)$. Find the product.

You may use small circles instead of small squares if you wish, but remember you are filling in area so it must occupy the same spaces as the length and width/height.

Think of matching columns and rows.

Distributive Property as Area: Whiteboards $2(3x+1)$

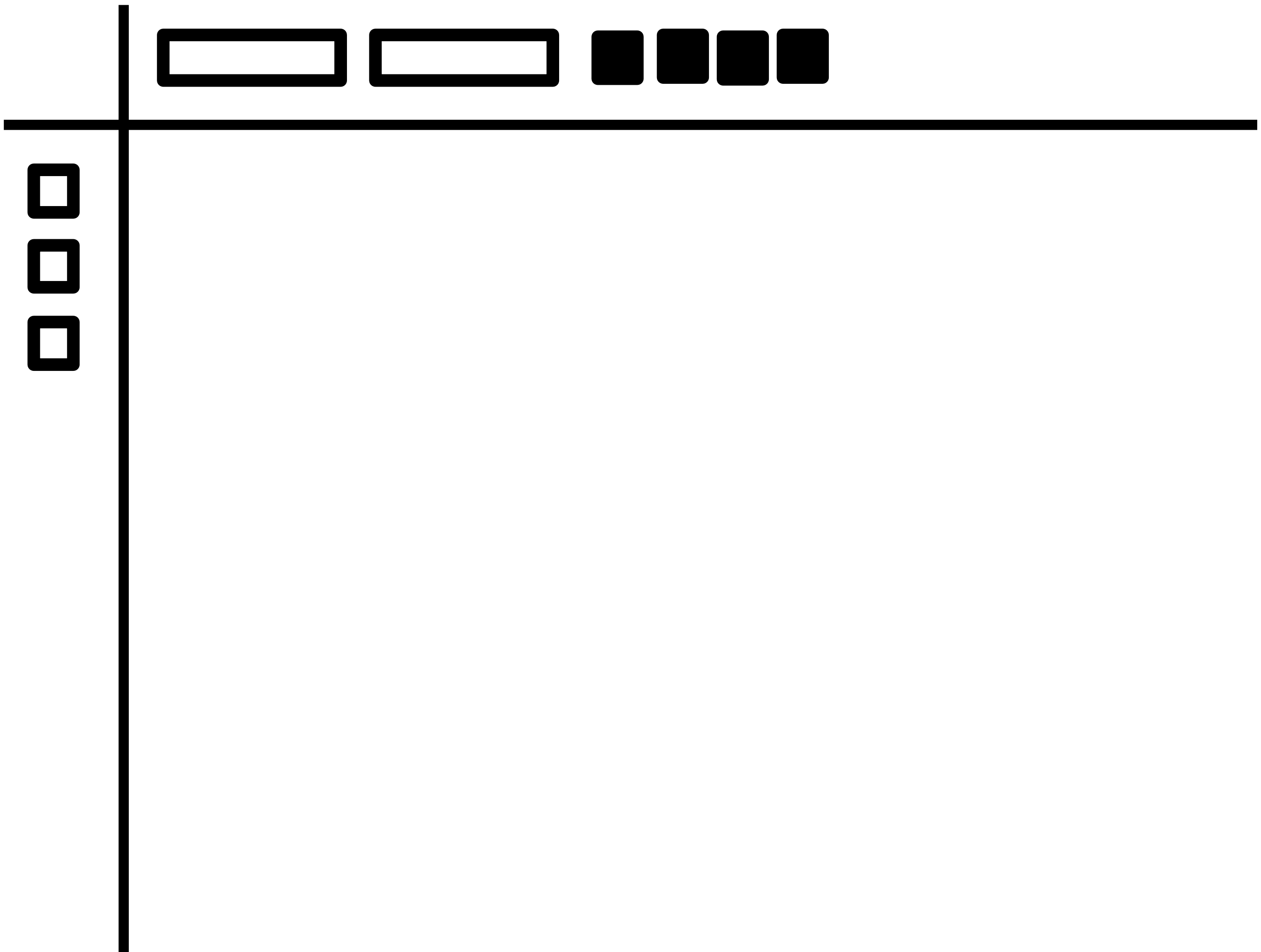


On your whiteboards, place tiles or draw $2(3x+1)$. Find the product.

Record the <u>product</u> in your four column note sheet.

You may use small circles instead of small squares if you wish, but remember you are filling in area so it must occupy the same spaces as the length and width/height.

Distributive Property as Area: Whiteboards

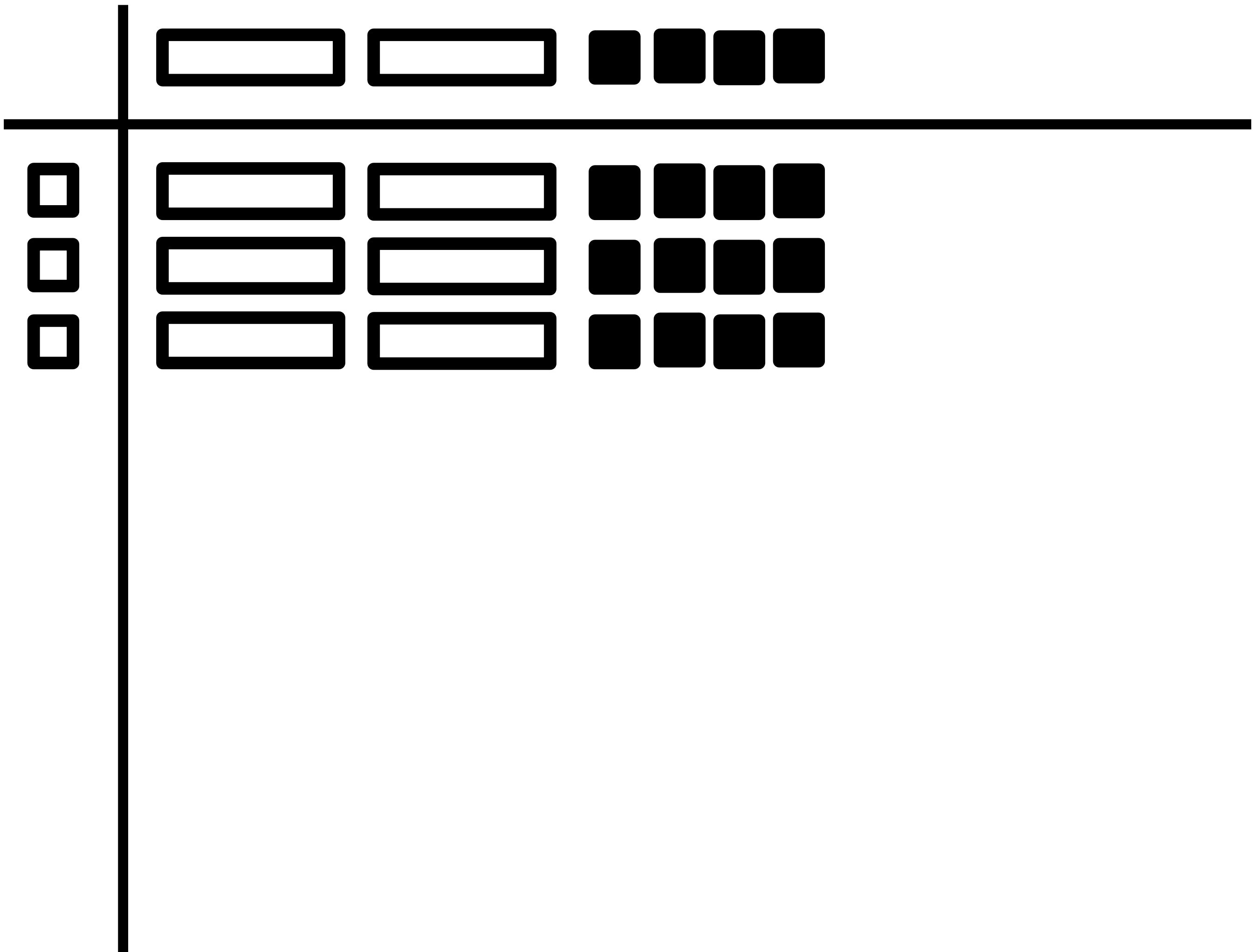


On your whiteboards, place tiles or draw $3(2x-4)$. Find the product.

You may use small circles instead of small squares if you wish, but remember you are filling in area so it must occupy the same spaces as the length and width/height.

Think of matching columns and rows.

Distributive Property as Area: Whiteboards

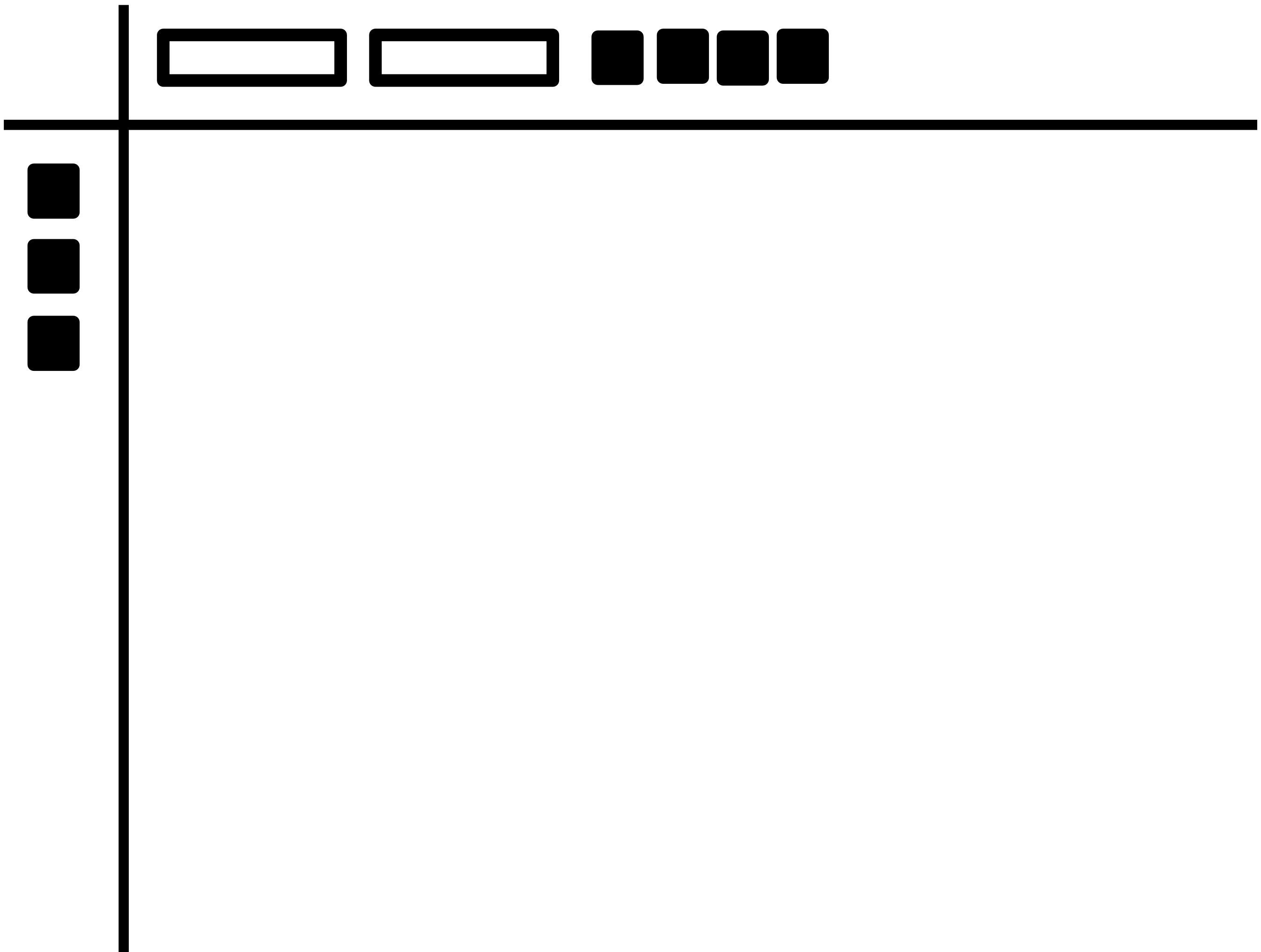


On your whiteboards, draw $3(2x - 4)$. Find the product.

Record the product in your four column note sheet.

What is the deal with negative area? It simply means that you would subtract off some of the area once you knew what the area generated by the base, x, was.

Distributive Property as Area: Whiteboards



On your whiteboards, place tiles or draw $-3(2x-4)$.

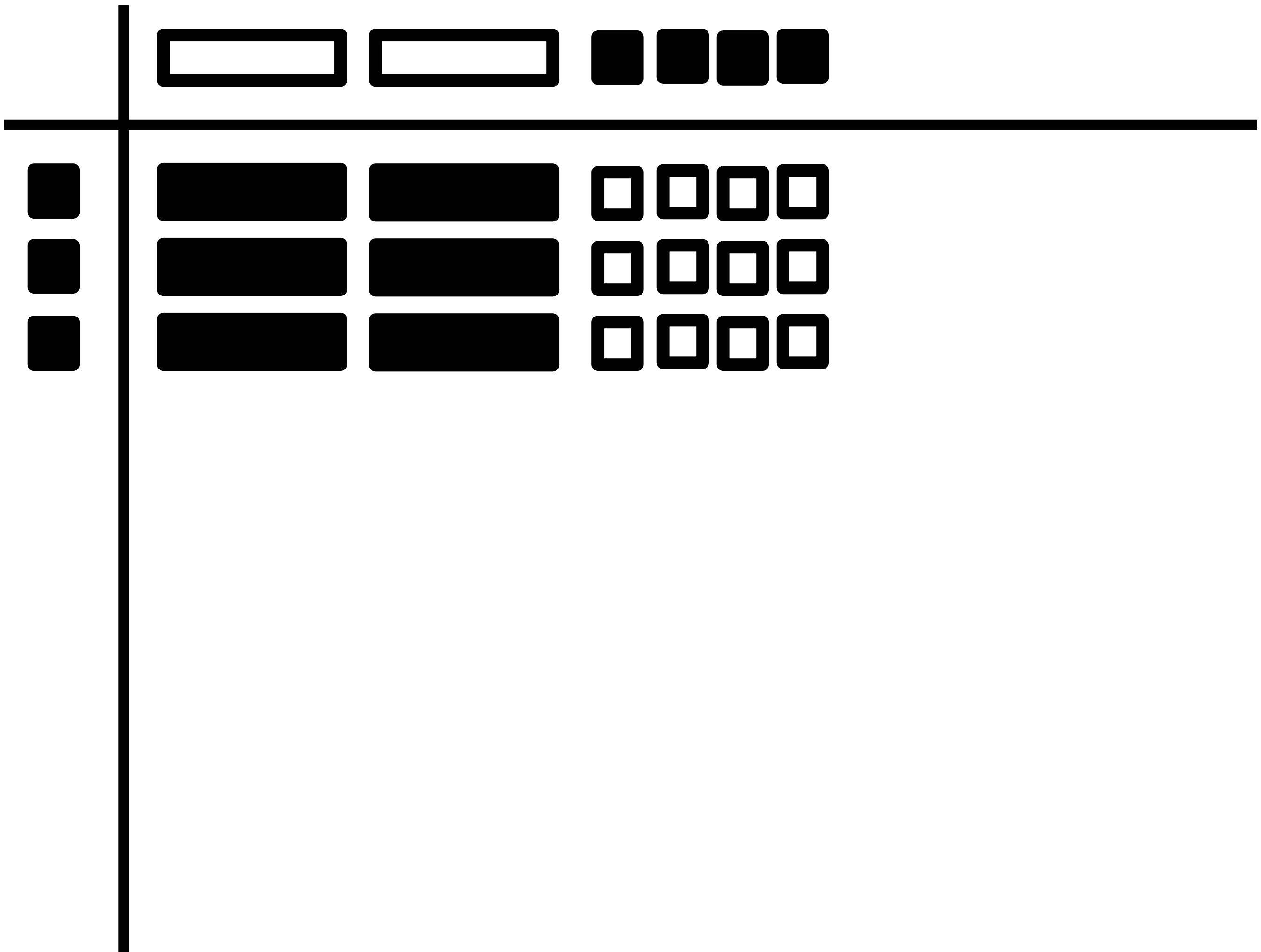
Find the product.

Remember that negative means opposite.

What two things did the negative three do the other terms?

Can you state this in terms of hot rocks and cold rocks? (If you used this context previously.)

Distributive Property as Area: Whiteboards



On your whiteboards, place tiles or draw $-3(2x-4)$.

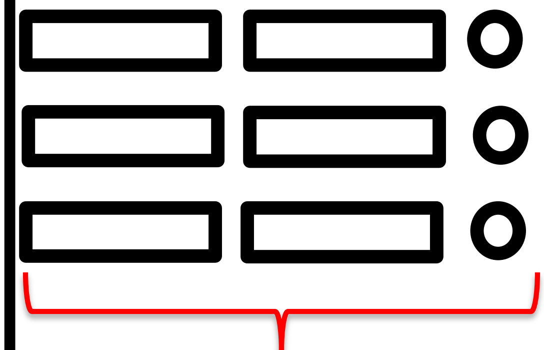
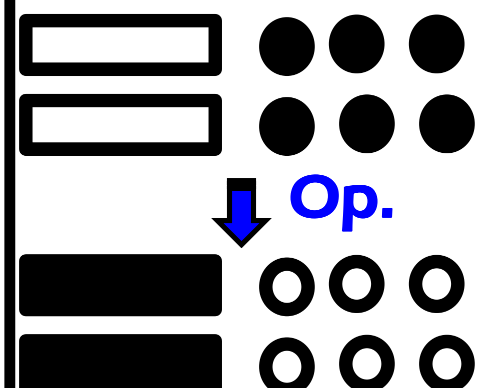
Find the product.

Record the product in your four column note sheet.

You remove three groups of 2x (hot rocks) which has the same effect as adding -6x (ice cubes.

You remove three groups of -4 (cold rocks), which has the same effect as adding 12 hot rocks.

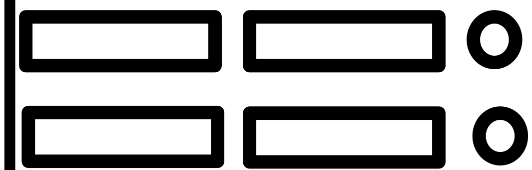
Distributive Property: Four Column Notes

Symbolic Expression	Pictorial Expression	Expanded Notation	Simplified Expression
$3(2x+1)$		$3(2x)+3(1)$	$6x+3$
Op. $-2(x-3)$			

Notice the negative. Read it as "the opposite of two groups of…"

Draw it before and after the opposite.

Distributive Property: Four Column Notes

Symbolic Expression	Pictorial Expression	Expanded Notation	Simplified Expression
			
		$3(x) + 3(2)$	

Notice this page is actually factoring.
How might doing it together help students?

Distributive Property as Area: Final Level Abstraction

Multiply each term within the parenthesis by the number of groups.
Remember to take the sign when you do!

$$7 - 5(2x - 3)$$

$$7 - 10x + 15$$

$$-10x + 8$$

After doing all this work, students should begin asking if they have to draw the shapes.

If not, then ask them if there is a way to go from

$$5(2x + 3)$$

to

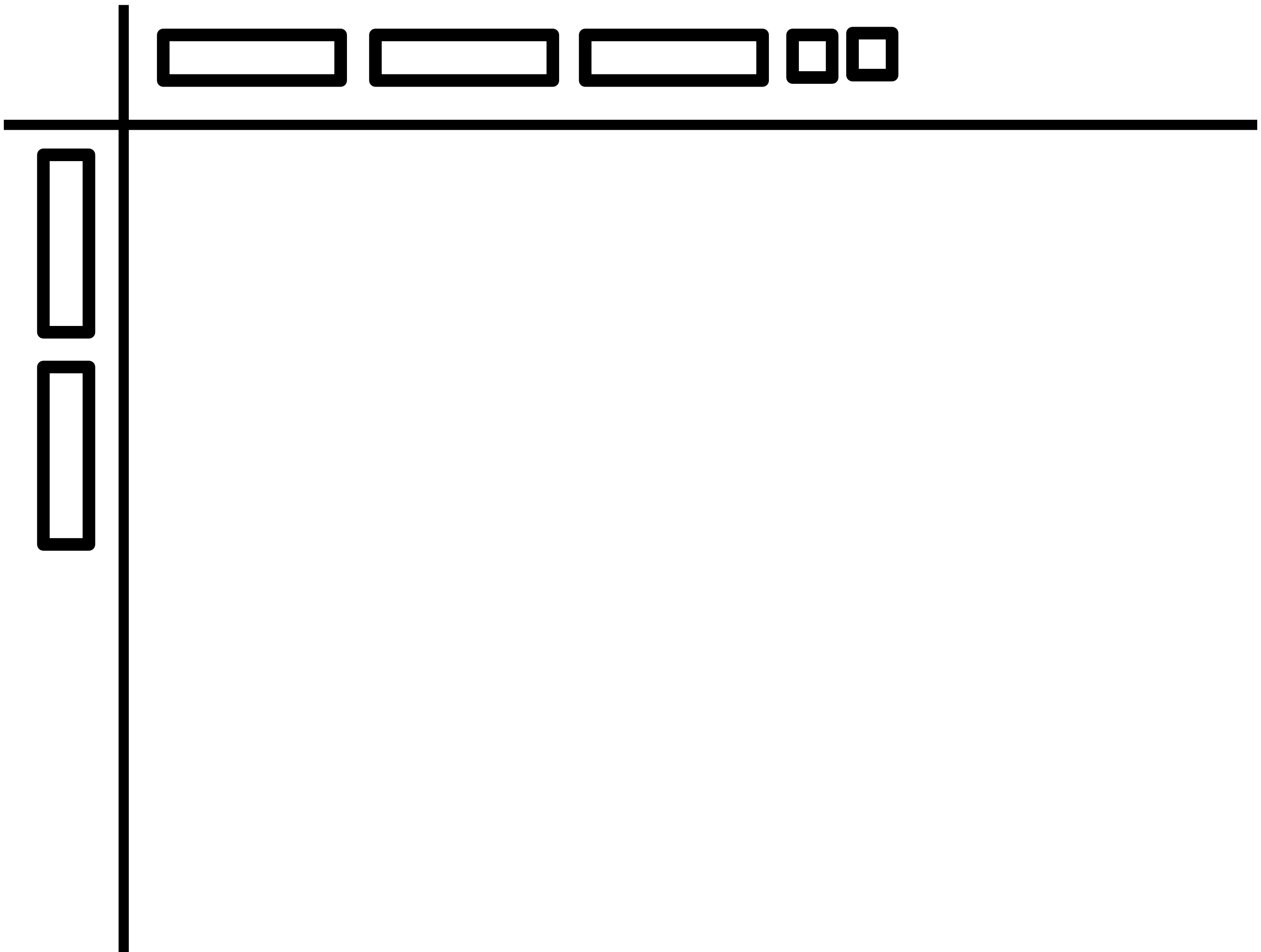
$$10x + 15$$

without drawing the shapes.

Mattoon's MEANING 4 MEMORY

Distributive Property as Area: Lesson Extension

Distributive Property as Area: Challenge Problem $2x(3x+2)$



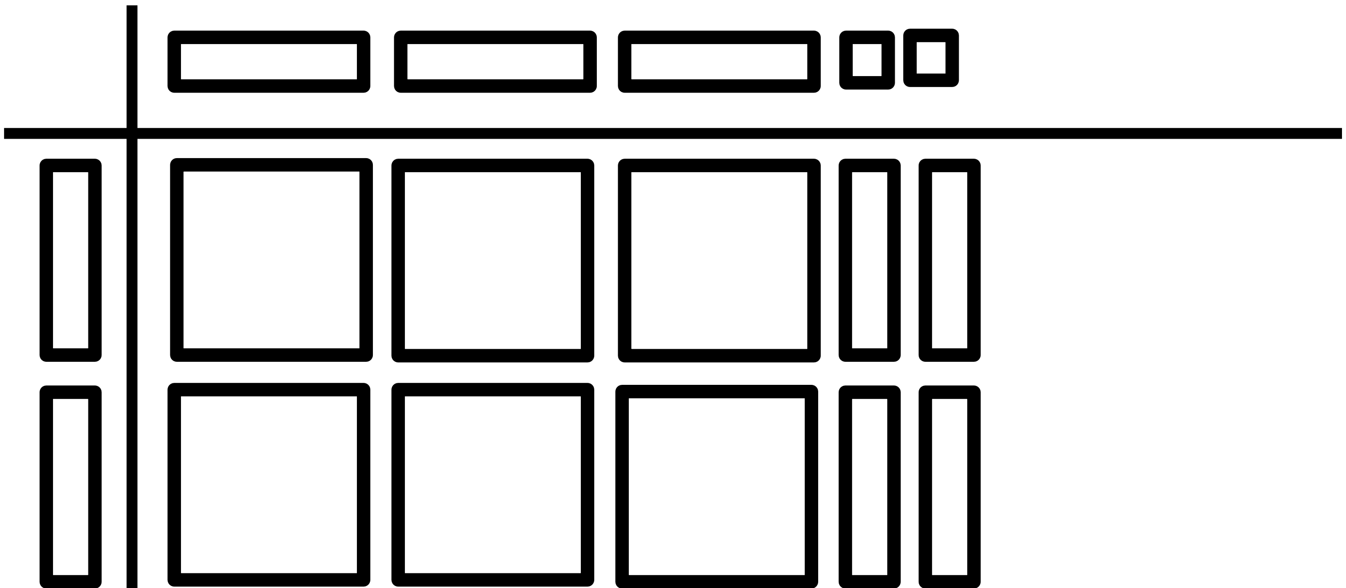
On your whiteboards, place tiles or draw $2x(3x+2)$. Find the product.

You may use small circles instead of small squares if you wish, but remember you are filling in area so it must occupy the same spaces as the length and width/height.

Think of matching columns and rows.

Distributive Property as Area: Challenge Problem $2x(3x+2)$

Mattoon's MEANING 4 MEMORY



$$6x^2 + 4x$$

On your whiteboards, place tiles or draw $2x(3x+2)$. Find the product.

Record the <u>product</u> in your four column note sheet.

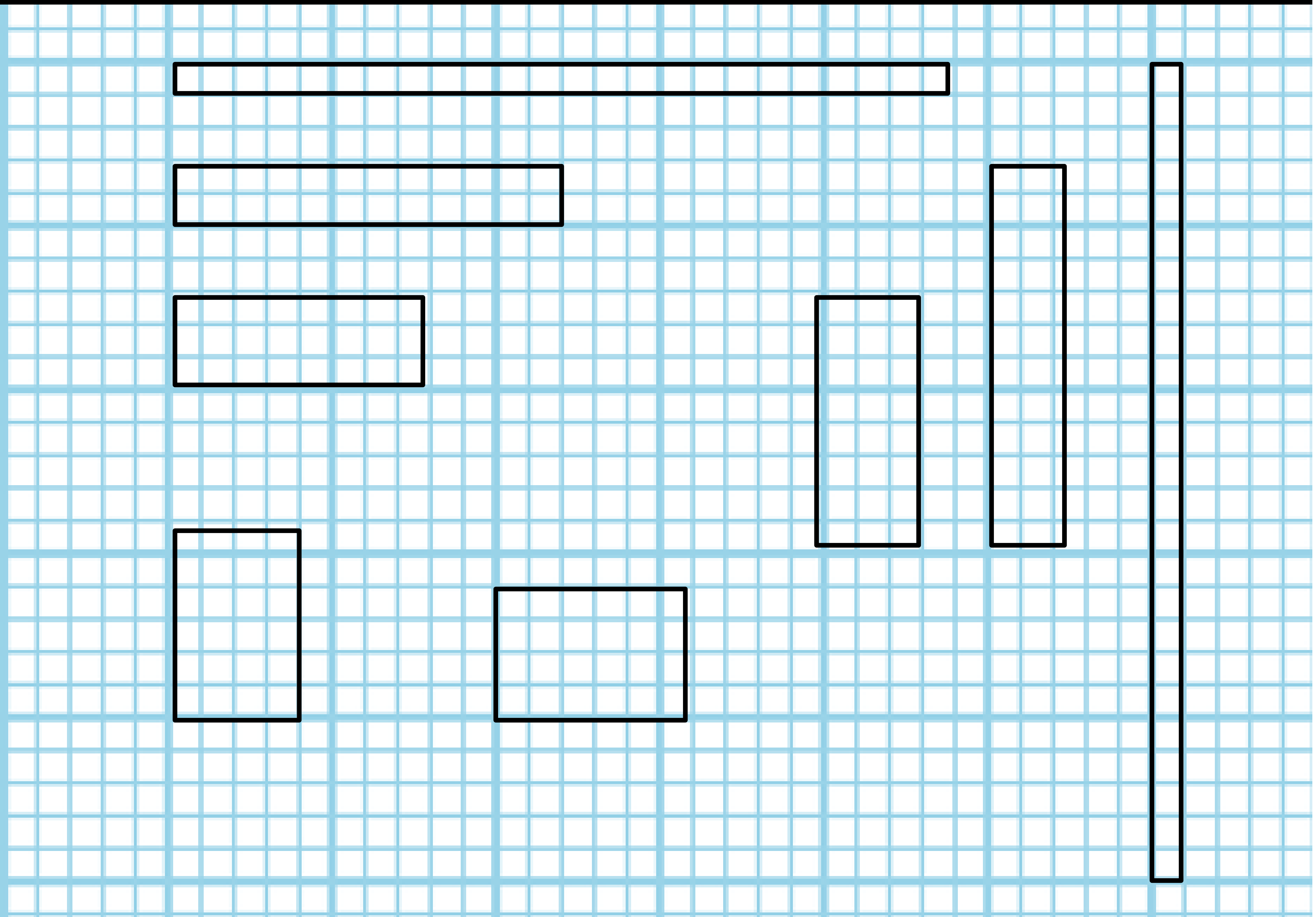
You may use small circles instead of small squares if you wish, but remember you are filling in area so it must occupy the same spaces as the length and width/height.

Factoring Linear Expressions: Finding Side Lengths

Lesson Introduction: Make Rectangles

On the gridded side of your whiteboard, make as many rectangles as you can with an area of 24 square units.

Lesson Introduction: Make Rectangles



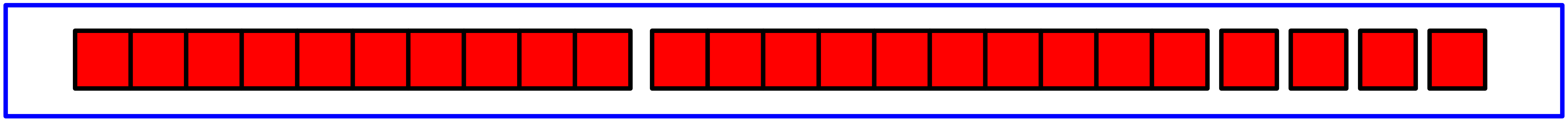
1 x 24
24 x 1
2 x 12
12 x 2
3 x 8
8 x 3
4 x 6
6 x 4

Discuss the Commutative Property here we will only draw one of each shape from here out.

Factoring with Base Ten Tiles

Without breaking up the 10 tile,

make another rectangle out of 24.



Any linear expression can be put into a rectangle with a height of one.

When asked to factor, we usually disregard this as an answer as it fails to break up the remaining term any further.

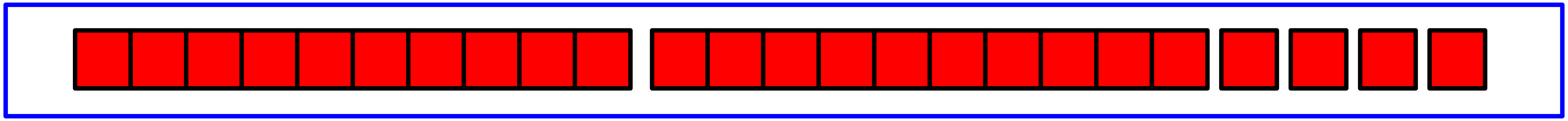
Write your rectangle in terms of area as an equation like

$$A = l \cdot w$$

Factoring 24 with Base Ten Tiles

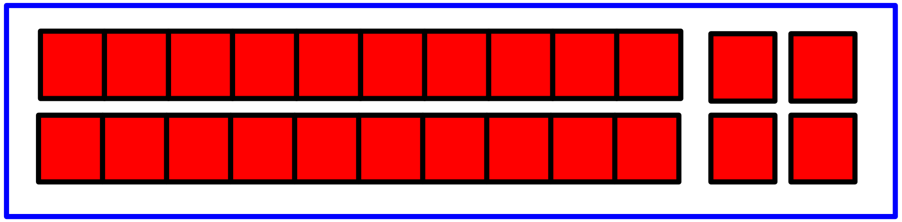
Mattoon's MEANING 4 MEMORY

Without breaking up the 10 tile, make another rectangle out of 24.



Any linear expression can be put into a rectangle with a height of one.

When asked to factor, we usually disregard this as an answer as it fails to break up the remaining term any further.



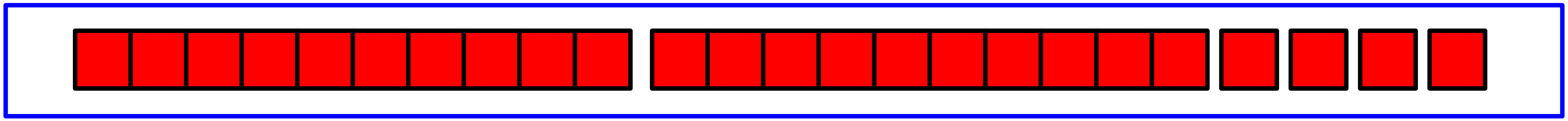
What is the length of this new rectangle?

What is the width, or height, of this new rectangle?

Can you see it on both sides? $A = l \cdot w$

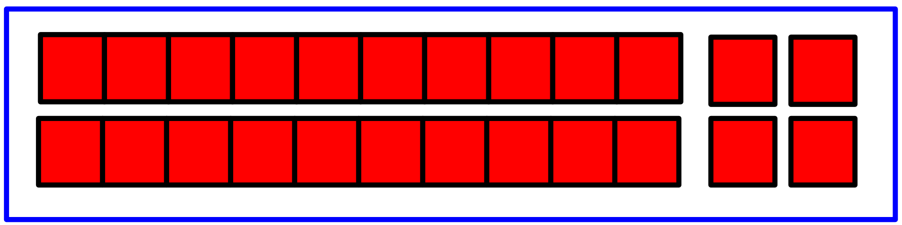
Factoring: Finding Side Lengths

Without breaking up the 10 tile, make another rectangle out of 24.



Any linear expression can be put into a rectangle with a height of one.

When asked to factor, we usually disregard this as an answer as it fails to simplify the remaining term further.



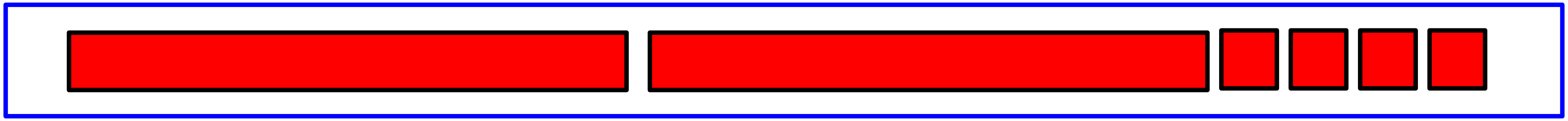
What is the length of this new rectangle? 12

What is the width, or height, of this new rectangle? 2

Can you see it on both sides? Yes $A = l \cdot w$ $24 = 12 \cdot 2$

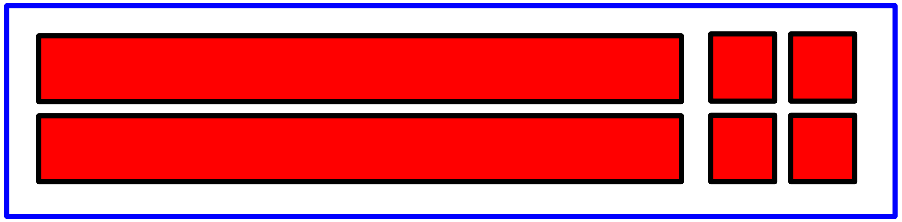
Factoring: Turn the linear terms over

Without breaking up the x tile, make another rectangle out of $2x + 4$.



Any linear expression can be put into a rectangle with a height of one.

When asked to factor, we usually disregard this as an answer as it fails to simplify the remaining term further.



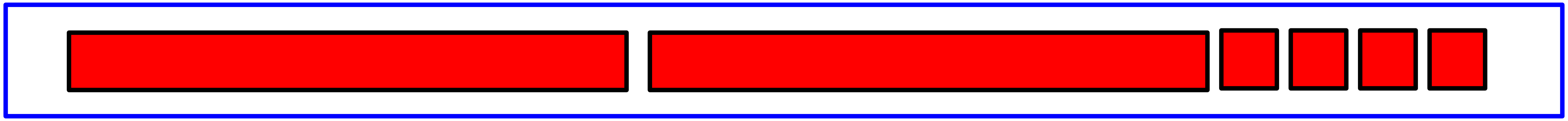
What is the length of this new rectangle?

What is the width, or height, of this new rectangle?

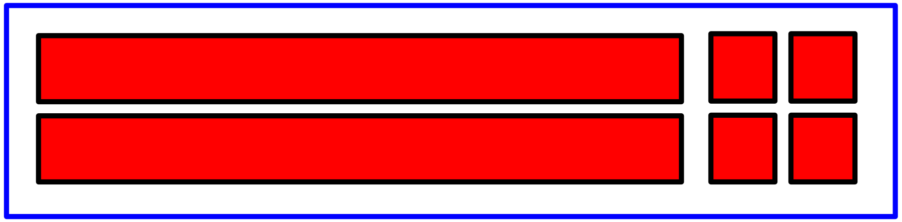
Can you see it on both sides? As, $A = l \cdot w$?

Factoring: Turn the linear terms over

Without breaking up the x tile, make another rectangle out of $2x + 4$.



Any linear expression can be put into a rectangle with a height of one. When asked to factor, we usually disregard this as an answer as it fails to simplify the remaining term further.



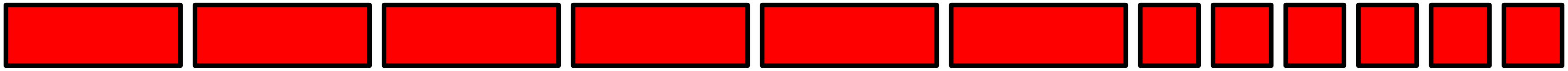
What is the length of this new rectangle? $1x + 2$

What is the width, or height, of this new rectangle? **2**

Can you see it on both sides? **Yes**

As, $A = l \cdot w$**?** $2x + 4 = 2(x + 2)$

Try Factoring $6x + 6$



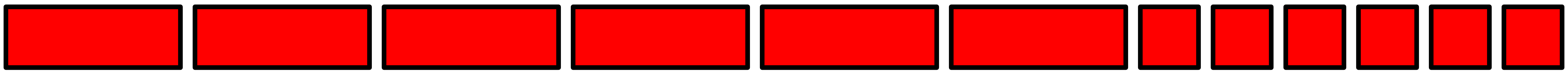
On the gridded side of your whiteboard, make as many rectangles as you can with

$$6x + 6$$

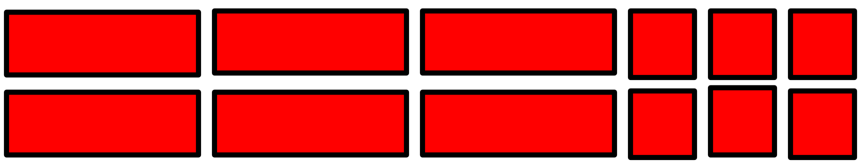
Label the length and width by writing a factored expression.

For example, the one above is $1(6x + 6)$

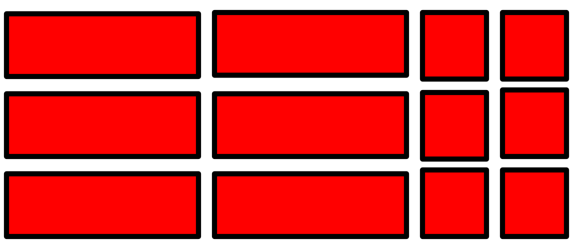
Try Factoring $6x + 6$, Common Factors



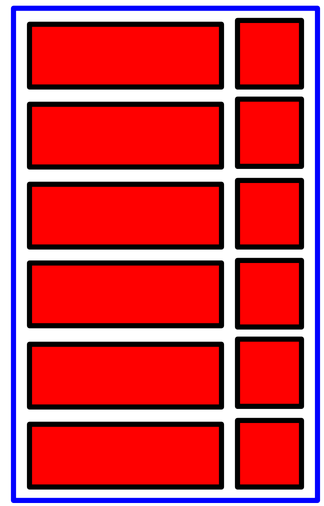
$1(6x + 6)$ **Like mentioned previously, we disregard this one as it fails to change at all.**



$2(3x + 3)$



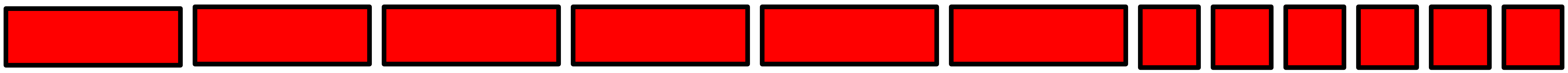
$3(2x + 2)$



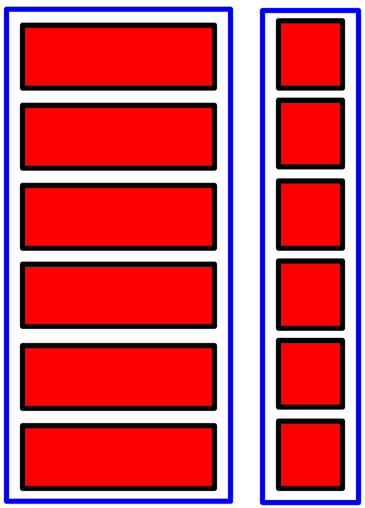
$6(x + 1)$

We say this one is factored completely as it has the most groups possible noted in front. This is called the Greatest Common Factor; whereas, the others are simply common factors.

Factoring $6x + 6$, Finding the GCF



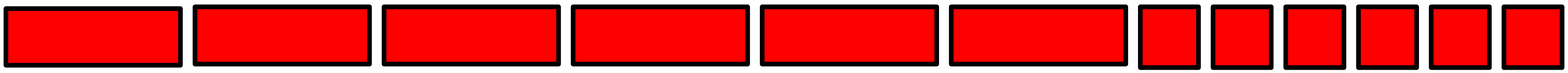
$6x + 6$



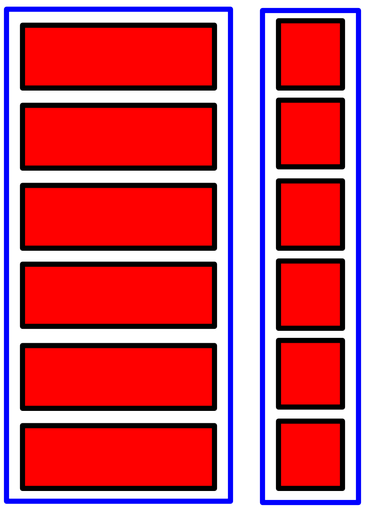
$6(x) + 6(1)$

6 is called the **Greatest Common Factor (GCF)** between 6x and 6. Both 6x and 6 can be made into 6 groups.

Factoring $6x + 6$, Distributing Backwards

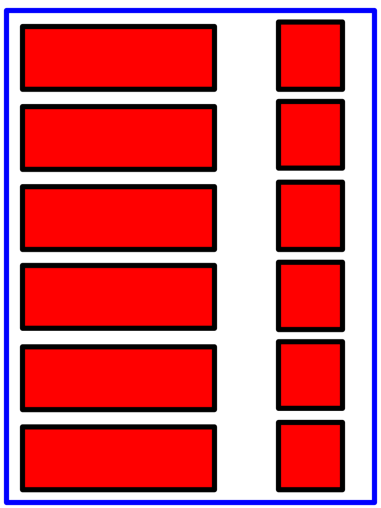


$6x + 6$



$6(x) + 6(1)$

6 is called the **Greatest Common Factor (GCF)** between 6x and 6. Both 6x and 6 can be made into 6 groups.



$6(x + 1)$

$6x + 6$

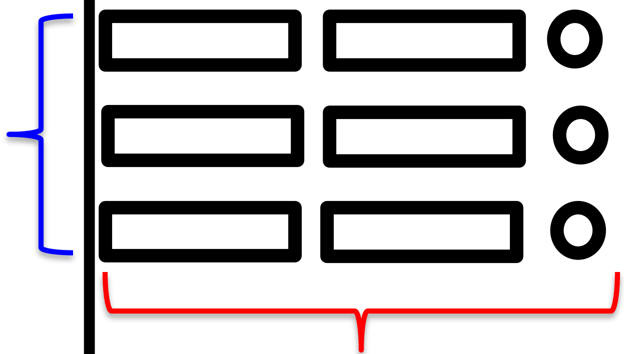
$6(x) + 6(1)$

$6(x + 1)$

Look familiar?

Factoring is like distributing backwards!

Distributive Property: Four Column Notes

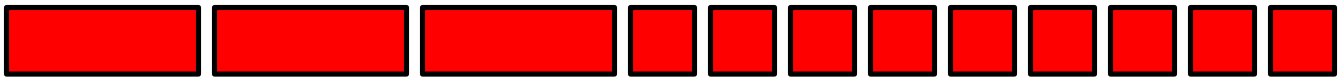
Standard Form (Given)	Pictorial Expression	Expanded Notation (Showing Factors of Each)	Factored Expression
$6x + 3$		$3(2x) + 3(1)$	$3(2x + 1)$

Why aren't we drawing the factors?

They are the length of the sides so they are already there.

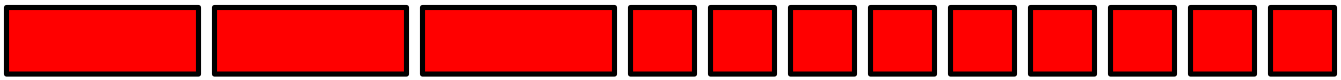
Try Factoring $3x + 9$
(Make a different rectangle)

Mattoon's MEANING 4 MEMORY

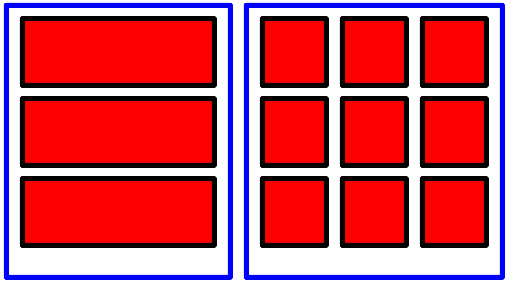


$3x + 9$

Factoring $3x + 9$, Find the GCF



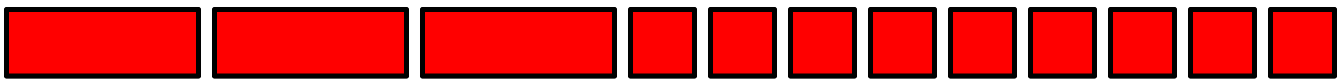
$3x + 9$



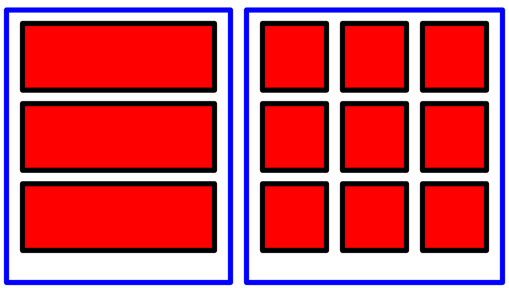
$3(x) + 3(3)$

The **Greatest Common Factor** between 3x and 9 is 3 as they both can be put into 3 groups .

Factor $3x + 9$, Distributing Backwards

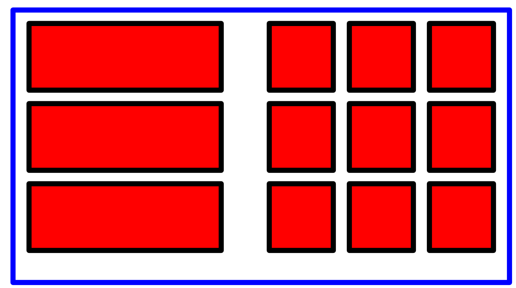


$3x + 9$



$3(x) + 3(3)$

The **Greatest Common Factor** between 3x and 9 is 3 as they both can be put into 3 groups .



$3(x + 3)$

$3x + 9$

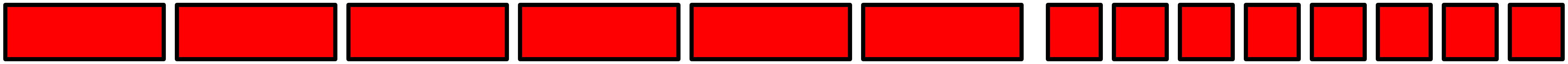
$3(x) + 3(3)$

$3(x + 3)$

Look familiar?

Factoring is like distributing backwards!

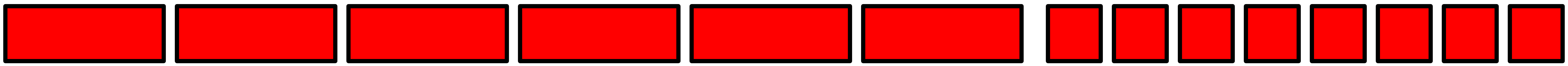
Try Factoring $6x + 8$
(Make a different rectangle)



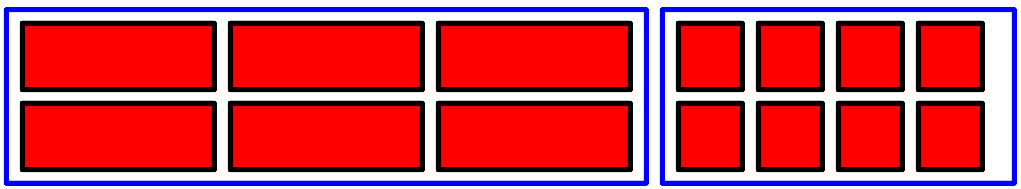
$6x + 8$

Factoring $6x + 8$, Find the GCF

MEANING 4 MEMORY



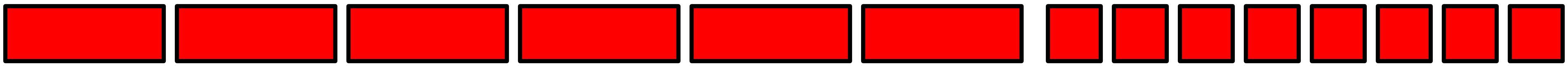
$6x + 8$



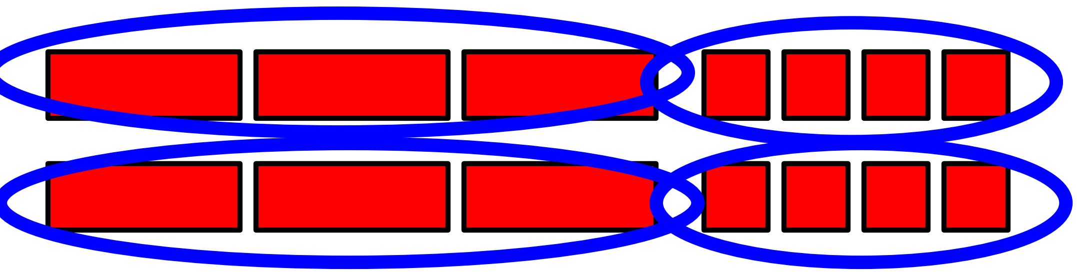
$2(3x) + 2(4)$

The Greatest Common Factor between 6x and 8 is 2 as they both can be put into 2 groups .

Factoring $6x + 8$, Distributing Backwards



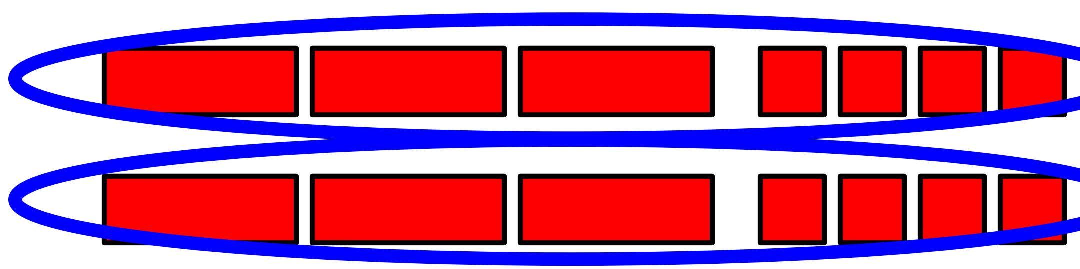
$6x + 8$



$2(3x) + 2(4)$

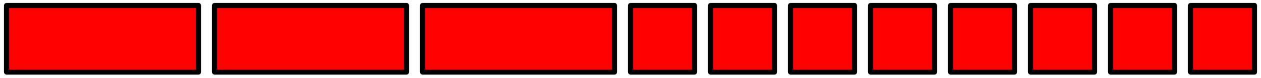
The **Greatest Common Factor** between 6x and 8 is 2 as they both can be put into 2 groups .

$$6x + 8$$
$$2(3x) + 2(4)$$
$$2(3x + 4)$$



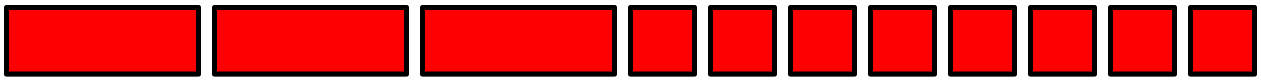
$2(3x + 4)$

Try Factoring $3x + 8$
(Make a different rectangle)

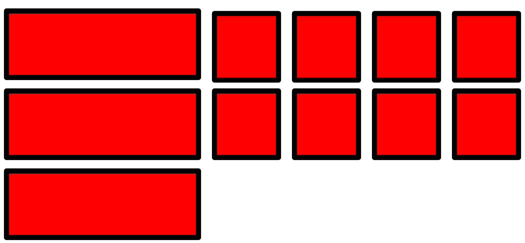


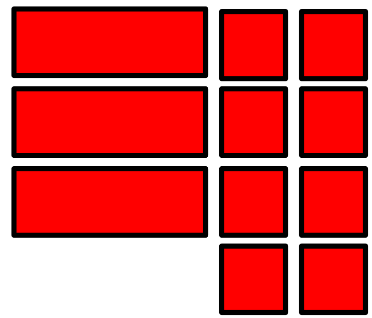
$3x + 8$

Try Factoring $3x + 8$ (Make a different rectangle)



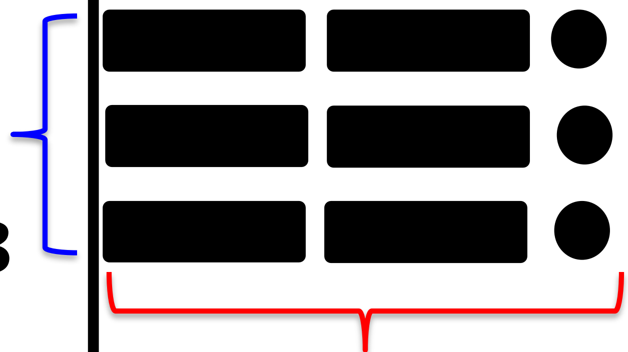
$3x + 8$





**The two terms share no common way to make a rectangle.
The two terms have no common factor besides one.
We say that this means the expression 3x+8 is not factorable or prime, the only factors are itself and one.**

Distributive Property: Four Column Notes

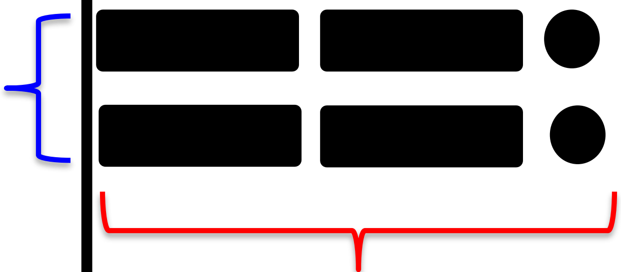
Standard Form (Given)	Pictorial Expression	Expanded Notation (Showing Factors of Each)	Factored Expression
$-6x-3$		$-3(2x)-3(1)$	$-3(2x+1)$
$-5x-10$			

If the negative is common to both, then you can factor it out.

The invisible + sign remains. See (2x+1) above.

Recheck your work using the distributive property.

Distributive Property: Four Column Notes

Standard Form (Given)	Pictorial Expression 	Expanded Notation (Shows Factors)	Factored Expression
$4x - 12$			

If the negative is common to both, then you can factor it out.

The invisible + sign remains. See (2x+1) above.

Recheck your work using the distributive property.

Final Level of Abstraction
Factor each term looking for the GCF

$$15x + 20$$

Given

$$5(3x) + 5(4)$$

Look for the Greatest Common Factor, the most amount of groups that can be made that are common to both or the longest possible side length common to both.

$$5(3x + 4)$$

Rewrite the two rectangles as a single rectangle, factor out the 5.

Final Level of Abstraction
Factor each term looking for the GCF

$$-\frac{1}{4}x+\frac{1}{2}$$

Given: factor out the coefficient.

Seventh versus sixth again.
Factor each term.

$$-\frac{1}{4}(1x)-\frac{1}{4}(-2)$$

How many ¼'s are in ½ ?
Watch your signs!

$$-\frac{1}{4}(x-2)$$

Write it as a product of two factors rather than addition/subtraction of two terms. Factor out the -1/4.

What Materials Did I Use?

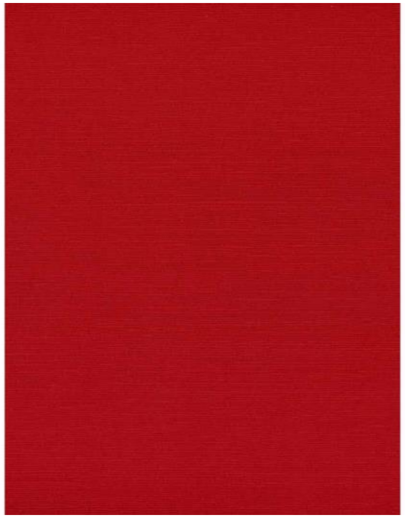
Two colors of cardstock

Slider bags

Folded blank paper

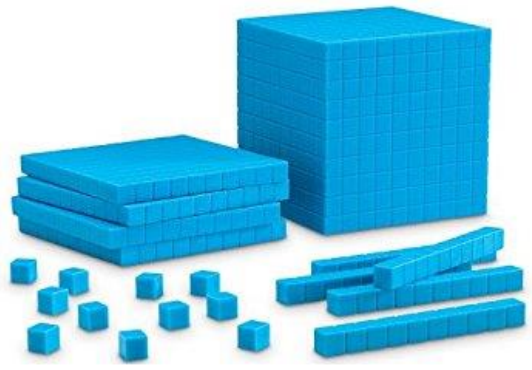
Cheapest manipulatives EVER!

And they do much more than this…



What about the Base Ten Blocks?

Some might argue against having the same manipulative as base ten and algebra tiles.



My counter argument is why keep two manipulatives when one will do?

More importantly, I *want* them to make connections between base ten and algebraic representations of any base to aid in understanding and retention.

Color and Algebra Tiles?

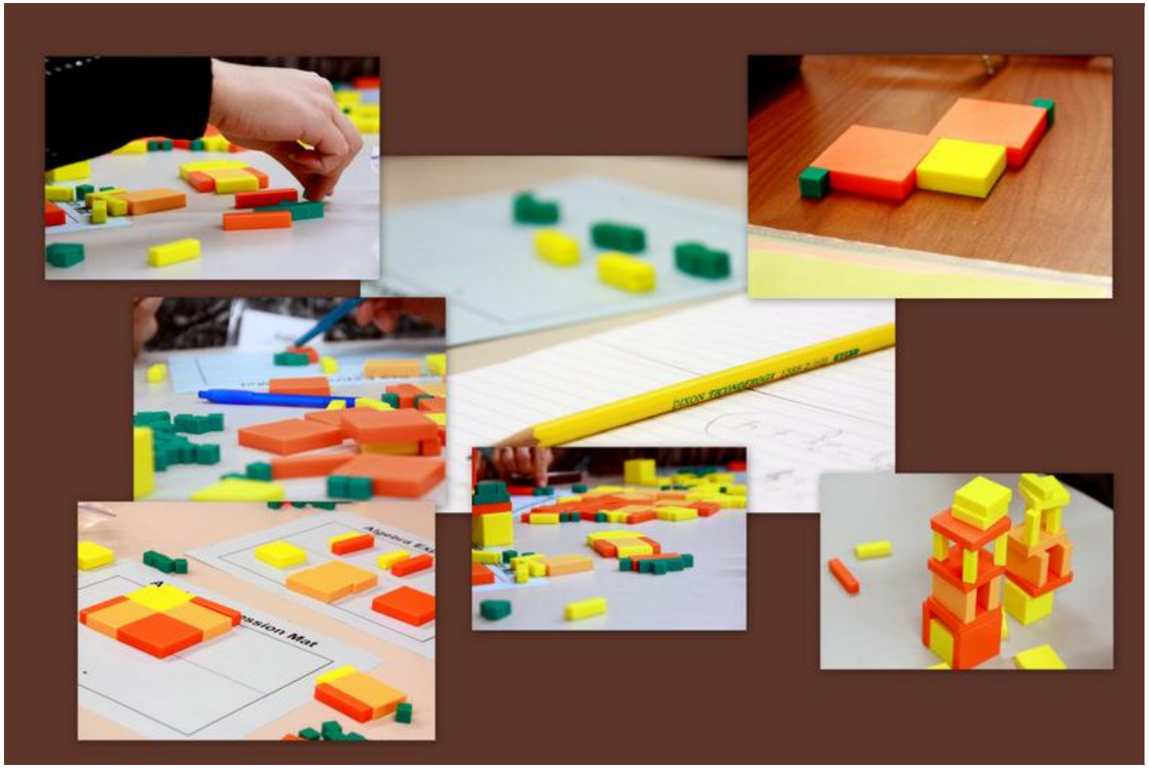
Store bought algebra tiles have "unknown" side lengths as they are trying to reinforce that quality of the variable.

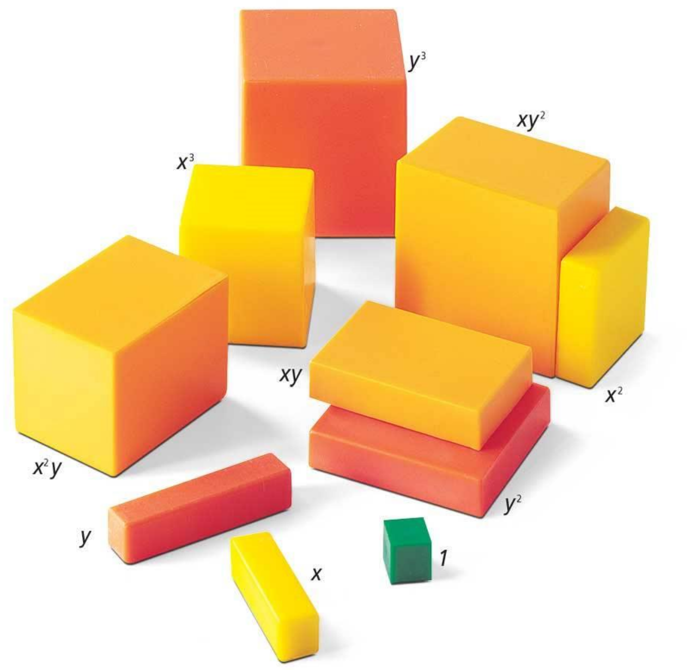
If you measure these tiles with the one, then you find they are usually between three and four. Does that mean it has a fractional side length?

All models have inherent flaws; choose the flaws that reinforce the learning when you can.

I don't prefer the multiple color tiles. I prefer to use color to distinguish between positive and negative or different variables.

What about Y? Color & Algeblocks





AlgeBlocks allow for work in three dimensions, cubic expressions & equations, and work with two variables, x & y.

http://www.hand2mind.com/item/algeblocks-manipulative-starter-set/9241 Expensive @ $200 though.

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